Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 233 MAY, 1942 No. 5

MOLDING THE CONSTITUTION.*

BY

GEORGE WHARTON PEPPER, ESQ.

It is timely, I think, to drive home the point that it was what Franklin *is* that enabled him to accomplish what he did rather than the specific proposals which he made or the instrumentalities which he employed to attain his ends. Franklin, as I understand him, is Personality in Action; and personality, I take it, is the secret of all real influence.

In this respect Franklin does not stand alone. There are at least two other great men to keep him company. It seems to me that the greatest contribution to history made by Washington, Lincoln and Franklin was the contribution of themselves. Other statesmen have accomplished equally great results by diplomacy, by eloquence, by liberal promises, by a thousand devices for securing popular support; but these men when their country's line was wavering brought up reserves of character, threw them into the breach, repelled the assault and turned defeat into victory.

It is difficult to exaggerate the importance of the point just made. We become so accustomed to measuring public men by what they say and the way they say it that we forget to study the man behind the words to determine whether

---

* Delivered in the Hall of The Franklin Institute, February 2, 1940. This was one of the papers in the "Meet Dr. Franklin" Conference.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

he is steadfast, unselfish and humble. Kipling's poem "If—" gives the specifications for the character of a real man; and there is good reason to believe that it was Washington's character that inspired the poem. The figure of Lincoln as it emerges from Sandburg's volumes is the figure of a man so steadfast, so selfless and so humble that you realize (perhaps for the first time) that in the inspired lines of the Second Inaugural you have the unconscious self-portrait of a man of God.

As I study Franklin's life the significance of his personality impresses me more and more. You may make a catalogue of all his inventions, a compilation of all his writings and a list of all his wise and witty sayings and you will scarcely have begun to understand the secret of his influence. The simple fact is that he was steadfast, intellectually honest, free from petty jealousy and malice and earnestly devoted to making the world a happier place for everybody to live in. The youth who would emulate Franklin should not attempt merely to imitate the things he did but to develop the homely virtues of which he was the embodiment. His diplomacy is the diplomacy of honor. His autobiography is a manifestation of candor. His office-holding is a record of unselfishness. His treatment of his enemies is free from malice. His service to his fellowmen is an application of the Golden Rule.

Coming specifically to the subject of this evening's conference, I suggest that Franklin's service in the Constitutional Convention was rendered rather by molding his colleagues than by any direct action in molding the Constitution itself. In other words, Franklin in the Convention is a problem of psychology rather than a study in political science.

To understand what happened in Independence Hall during the summer of 1787 you must have in mind the course of world events during the preceding thirty-three years of the XVIII century. The Constitutional Convention and what it accomplished was not a sudden and miraculous occurrence. It was merely a significant stage in the long evolutionary process which resulted in the birth of a nation.

A glance at the chart which I have prepared and put into your hands will refresh your recollection of some significant XVIII Century events. You will note the life-lines of three groups of notables: (1) those born in the XVII Century and living into the XVIII; (2) those whose births and deaths occurred between 1700 and 1800, of which group Washington and Franklin are conspicuous members; and (3) those born before 1800, whose lives extended into the XIX Century. Next you will note the descent of the English Crown and of the French Crown and the origin and evolution of the Government of the United States. Then you will notice vertical lines indicating by their length the duration of some of the many wars which spatter the entire century with blood. Finally you will turn with relief to some of the events which, in spite of continual wars, made the XVIII Century notable in the history of letters, the arts and the sciences.

As to the wars, two things may be said: first that each of them, although European in origin, quickly gave rise to a related struggle in America; and, second, that the necessity for effective common defence supplied the people of the thirteen Colonies with the strongest motive for union.

The military necessity for colonial union was recognized by the English government as well as by the Colonists. In 1754 the Seven Years' War was imminent and at the suggestion of the Crown the Board of Trade called for a conference of commissioners from the several Colonies to be held at Albany for the two-fold purpose of retaining the friendship of the Indians of the Six Nations and placing the Colonies in a state of defence against the French. Seven of the Colonies (including Pennsylvania) sent commissioners to Albany and there the conference adopted the plan of colonial coöperation and union which Franklin had drafted. Not one of the Colonial legislatures approved it, as there was a jealous fear of intrusting to a central council as much power as the plan wisely provided for. Not having received colonial assent, the plan was never formally submitted to the Crown. There is good reason, however, to believe that the Crown would have disapproved it for a contrary reason—namely that as between the Colonies and the Mother Country it savored too much of the independence of the former. Franklin shrewdly observed that the different and contrary reasons for disapproving his plan indicated that it was in fact the true medium. Had the plan been adopted the Colonies would have been able to defend themselves without the aid of troops from England; and thus the excuse for British taxation of the Colonies would never have arisen; and it is a fair speculation that the War of the Revolution itself might never have been fought. It will be remembered that the Albany plan, far from contemplating independence, was viewed by Franklin and his colleagues as a measure of imperial defence. He was working earnestly and sincerely to maintain the integrity of the British Empire.

Following the Seven Years' War, called on this side of the water the French and Indian War, the British Parliament began to assert a right of absolute legislative control over the Colonies. This manifested itself in the imposition of taxes and in other ways. Against these acts of parliamentary usurpation the Colonists vigorously protested. If as much heed had been paid to their protests as has in our own day resulted in according self-government to the several dominions within the British Empire the whole course of history would have been changed. In fact, however, the Crown elected to treat these colonial protests as acts of insubordination and disloyalty and by so doing caused the Colonists to become aware of a national unity not theretofore recognized. Accordingly on October 14, 1774, the First Continental Congress, in session at Philadelphia, adopted a Declaration of Resolves which spoke of maintaining peace but contained a strong hint of possible resistance by force. Pursuant to this action a petition to Parliament was prepared and sent to London in Franklin's care. Following its submission and rejection and the humiliating treatment to which he himself was subjected before the Privy Council he came home and was promptly elected to the Second Continental Congress.

In this Congress, which met in May of 1775, Franklin was by far the oldest member. From the records of its proceedings I gather that he at seventy played a like part and exercised a like influence to that which twelve years later he exercised in the Constitutional Convention. He was courageous and firm but sparing of words. Jefferson, who served with him, testifies that he never heard Franklin speak more than ten minutes but always to the main point. Like Washington, Franklin laid his shoulder to the great point knowing that the little ones would follow. He had not lost hope that the colonists might find liberty within the Empire; but his hope gradually faded. In May of '75 the celebrated "Mecklenburg Resolves" had been adopted in North Carolina. These treated the conduct of Parliament as having automatically annulled and vacated all royal authority in the Colonies and took appropriate action to set up local self-government until either Parliament should reverse itself or the Continental Congress should enact general legislation. Following this action the Congress on July 6, 1775, made a momentous declaration that as between submission to tyranny and an appeal to arms they had decided on the latter. The die had been cast and a declaration of independence became only a question of time.

Franklin's amazing activities during the remainder of 1775 and the following year even included organization for the defence of Philadelphia in the event of the war which was now inevitable. Events moved fast. In May of 1776 the Virginia Convention instructed the Virginia delegates to propose in Congress the long-expected declaration. This proposal was duly made by Richard Henry Lee. The resolution was adopted on July 2nd and a committee appointed to draft the document. Jefferson, acting in Lee's place when the latter was called home by his wife's illness, made the draft, embodying in it the principles and propositions which had been framing themselves in the popular mind and were constantly voiced in debates and speeches. It was perfected in conference with Franklin and John Adams, somewhat amended by Congress in the course of debate, adopted on July 4, 1776, published in the Pennsylvania *Evening Post* of July 6th and on Monday July 8th proclaimed to the people in the State House yard.

It remained to agree upon a framework of federal government. This was no easy task. Not until sixteen months later (November 15, 1777) did Congress agree upon Articles of Confederation and Perpetual Union; and it was not until March 1, 1781, that the thirteenth State (Maryland) ratified the Articles, thereby completing the act of confederation. The enthusiasm for independence had failed to unify the thinking either of representative statesmen or of the people of the several States.

Limitations of time forbid an analysis of the Articles. Two points, however, I must mention: one, that the members of Congress under the Articles were to be elected by the State legislatures; the other, that in determining questions in the United States in Congress assembled each State, whether large or small, was given one vote.

The new government was set up but, as you all know, it proved to lack central powers of control. Controversies as to trade and commerce were particularly acute; and on January 21, 1786, the General Assembly of Virginia resolved to appoint commissioners to meet at Annapolis with commissioners from other States to consider and report how far uniform commercial regulations may be necessary. Commissioners from only five States responded to this call, viz., New York, New Jersey, Pennsylvania, Delaware and Virginia. Franklin was not a commissioner. They, however, met and recommended not only to their several States but also to Congress that a convention of representatives of all the States be called to consider what changes in the Articles of Confederation may be necessary to make the federal government adequate to the exigencies of the Union. Congress on February 21, 1787, took favorable action and declared that "it is expedient that on the second Monday in May next a convention of delegates who shall have been appointed by the several States be held at Philadelphia for the sole and express purpose of revising the Articles of Confederation and reporting to Congress and the several legislatures such alterations and provisions therein as shall when agreed to in Congress and confirmed by the States render the federal constitution adequate to the exigencies of Government and the preservation of the Union."

The Convention, as you know, met in Independence Hall and sat behind closed doors. The total number of delegates chosen was 65, of whom 10 never attended. It was not until May 25th that a quorum of seven States could be secured. From then onward to and including September 17th the delegates remained in session, all of the thirteen States except Rhode Island being sooner or later represented. General Washington was unanimously chosen president. The motion for his election was to have been made by Franklin; but on the opening day he was prevented from attending. Thereafter, however, during the long hot summer his attendance was regular in spite of his more than eighty years and the painful malady which three years later was to prove fatal.

One of the delegates from Georgia, Major Pierce, made a series of thumb-nail sketches of many of his colleagues which are most entertaining. He gives the approximate age of each and his figures (excluding Franklin) yield an average age of less than forty. In other words, Franklin's age was more than double the average. The youngest man in the body was Charles Pinckney of South Carolina, who was twenty-four, and only three others were as old as sixty. I mention these statistics because there is a popular impression that the Framers were a body of hard-boiled old plutocrats with long grey beards.

Listen to what Major Pierce has to say about Franklin:

> Dr. Franklin is well known to be the greatest phylosopher of the present age;—all the operations of nature he seems to understand,—the very heavens obey him, and the Clouds yield up their Lightning to be imprisoned in his rod. But what claim he has to the politician, posterity must determine. It is certain that he does not shine much in public Council,—he is no Speaker, nor does he seem to let politics engage his attention. He is, however, a most extraordinary Man, and tells a story in a style more engaging than anything I ever heard. Let his Biographer finish his character. He is 82 years old, and possesses an activity of mind equal to a youth of 25 years of age.

It is extremely difficult for those who have not read the debates as recorded in Madison's notes to realize the seemingly hopeless differences of opinion which developed on every point brought forward for discussion. The old jealousies which a generation earlier had made the Albany plan unacceptable were still much in evidence. There were at least half a dozen different sorts of difficulties to contend with. First there were differences of opinion as to the powers of the Convention as a whole, i.e., whether the power to amend the Articles of Confederation justified the substitution of a substantially new instrument. Second, many of the State delegations felt themselves bound by limitations imposed upon them by the terms of their appointment; and in some instances (notably New York) the delegation was divided against itself as to the scope of these limitations.

Next there was the deep-seated prejudice of some in favor of a strong central government and an equally strong insistence by others upon the unlimited sovereignty of the States. Again there were sectional differences, as between New England and the States farther south and the seeming impasse between the views of large and small States on the question of representation. Add to all this the conflict of ideas on every conceivable part of the machinery of government and you have a situation in which nothing but unlimited patience and infinite tact could avert disaster.

As an illustration of the disputes over the mechanism of government I may mention the collision between the Virginia plan and the New Jersey plan. Speaking on the 16th of June, James Wilson summarized the differences between the two plans as follows:

> Virginia plan proposes two branches in the legislature.
> Jersey a single legislative body.
> Virginia, the legislative powers derived from the people.
> Jersey, from the States.
> Virginia, a single executive.
> Jersey, more than one.
> Virginia, a majority of the legislature can act.
> Jersey, a small minority can control.
> Virginia, the legislature can legislate on all national concerns.
> Jersey, only on limited objects.
> Virginia, legislature to negative all State laws.
> Jersey, giving power to the executive to compel obedience by force.
> Virginia, to remove the executive by impeachment.
> Jersey, on application of a majority of the States.
> Virginia, for the establishment of inferior judiciary tribunals.
> Jersey, no provision.

Whereupon, as if there were not trouble enough already, Hamilton advanced what he described as "insuperable objections" to both plans—and proceeded to propose a third. Thereafter the seemingly endless debates proceeded without arriving anywhere. Then on June 28th Franklin submitted his memorable remarks in support of a motion to open each session of the Convention with prayer. Said he:

> The small progress we have made after 4 or five weeks close attendance & continual reasonings with each other—our different sentiments on almost every question, several of the last producing as many noes as ays, is methinks a melancholy proof of the imperfection of the Human Understanding. We

> indeed seem to feel our own want of political wisdom, since we have been running about in search of it. We have gone back to ancient history for models of Government, and examined the different forms of those Republics which having been formed with the seeds of their own dissolution now no longer exist. And we have viewed Modern States all around Europe, but find none of their Constitutions suitable to our circumstances.
>
> In this situation of this Assembly, groping as it were in the dark to find political truth, and scarce able to distinguish it when presented to us, how has it happened, Sir, that we have not hitherto once thought of humbly applying to the Father of lights to illuminate our understandings? In the beginning of the Contest with G. Britain, when we were sensible of danger we had daily prayer in this room for the divine protection.—Our prayers, Sir, were heard, & they were graciously answered. All of us who were engaged in the struggle must have observed frequent instances of a superintending providence in our favor. To that kind providence we owe this happy opportunity of consulting in peace on the means of establishing our future national felicity. And have we now forgotten that powerful friend? or do we imagine that we no longer need his assistance? I have lived, Sir, a long time, and the longer I live, the more convincing proofs I see of this truth—*that God Governs in the affairs of men*. And if a sparrow cannot fall to the ground without his notice, is it probable that an empire can rise without his aid? We have been assured, Sir, in the sacred writings, that "except the Lord build the House they labour in vain that build it." I firmly believe this; and I also believe that without his concurring aid we shall succeed in this political building no better, than the Builders of Babel: We shall be divided by our little partial local interests; our projects will be confounded, and we ourselves shall become a reproach and bye word down to future ages. And what is worse, mankind may hereafter from this unfortunate instance, despair of establishing Governments by Human wisdom and leave it to chance, war and conquest.
>
> I therefore beg leave to move—that henceforth prayers imploring the assistance of Heaven, and its blessings on our deliberations, be held in this Assembly every morning before we proceed to business, and that one or more of the Clergy of this City be requested to officiate in that Service—

233
42

No formal action was taken on this motion but the old man's confession of faith doubtless impressed his colleagues as much as it has posterity. As I study the debates I fail to find that any delegate other than Franklin was really eager to reach any solution except his own solution. Each speaker had a view of his own and was determined that the country should be saved in his way or not at all. Franklin evidently realized that even the most confident proposals were really nothing but experimental suggestions and that the important thing was to secure unity of action. Others had what they regarded as convictions and from time to time spoke of being restrained by their consciences. It might be said of each of several of such delegates what a friend of mine recently wrote me about another friend. "He seems" ran the letter "always to stand in his own light by an ardent passion for righteousness which suffuses even the most mundane issues." Franklin knew that debatable questions of political science were not fit subjects for conscientious conviction. While some of his colleagues (Randolph of Virginia, for example) raised moral issues the wise old Franklin dealt with them all as fit subjects for accommodation and adjustment.

He had his preferences—some of them most decided. He was opposed to a property qualification for legislators and thought that electors need not be freeholders. He did not think that the elected had any right in any case to narrow the privileges of the electors. He favored the impeachability of the Executive, whom he regarded as servant of the people. He viewed executive power with suspicion. Referring to the experience of the United Netherlands he reviewed the executive encroachments of the Prince of Orange and the efforts of a later Stadtholder to make the office hereditary. He also cited the example of Pennsylvania where he, Franklin, had had an unhappy experience with the Proprietors. "The first man put at the helm" said he, "will be a good one. Nobody knows what sort may come afterwards. The Executive will be always increasing here as elsewhere until it ends in a monarchy."

Another suggestion was that public servants should be unsalaried. At one point I find this entry in Madison's notes: "The 3d Resolution 'that the national Legislature ought to consist of two branches' was agreed to without debate or dissent, except that of Pennsylvania, given probably from complaisance to Doc^{r}. Franklin who was understood to be partial to a single House of Legislation." He opposed the requirement of a long term of citizenship for senators. When the appointment of federal judges was under consideration Franklin instanced the fifteen Lords of Sessions in Scotland who were appointed, he said by the barristers or doctors. I can imagine the twinkle in his eye when he added: "These elect the most learned of their own order, because he has the most business which afterwards is divided among themselves."

He favored the provision forbidding the diminution of judicial salaries as applied to men in office, but thought it wise to permit an increase which, he observed, would not create any improper dependence in the judges. He favored the requirement of two witnesses to the same overt act of treason. "Prosecutions for treason" he said "were generally virulent and perjury too easily made use of against innocence."

The most dramatic moment in the Convention was that when the controversy between large and small States over the question of representation had threatened to disrupt the Assembly and put an end to all hope of agreement. Nothing but a careful reading of the debates can give an adequate idea of the seriousness of this emergency. Opinions were more definite and seemingly more irreconcilable on this question than on any other. It was at this point that Franklin rendered his greatest service. "There ought to be" he said, "some difference between the first and second branches. Many expedients have been proposed and, I am sorry to remark, without effect. A joiner, when he wants to fit two boards, takes off with his plane the uneven parts from each side and thus they fit. Let us do the same—we are all met to do something." He proceeded with his proposal that in the Senate the States should have equality of representation, thus leaving the composition of the other House to be determined by representation proportionate to population. The compromise proposal was bitterly resisted. A committee was appointed to consider the subject and report. The committee made Franklin's motion the basis of its report but the report itself was vehemently opposed by a number of delegates including both Madison and Wilson. Gradually, however, the expediency of the proposal wore down the opposition. The provision that money bills (i.e. bills for raising revenue) should originate in the more numerous branch of the legislature where the large States held the majority was recognized as a concession by the smaller States. Even after the committee's report had been accepted and Franklin's proposal had thus been translated into action the opposition died hard. A proposal was made to recede from the compromise and was defeated by a vote in which the ayes were 4 and the noes were 6, with one State divided. "This question was taken" Rufus

King of Massachusetts reports in his notes, "and to my mortification by the vote of Mass. lost."

This significant incident took place in mid-July. As I read the debates during the remainder of the session, I sense a tendency to agree in contrast to the earlier tendency to disagree. I suspect that Franklin's moderating influence had extended beyond the immediate issue and that even the most opinionated of the debaters had come to realize that compromise was possible all along the line. Gradually the governmental structure as we now know it took form. A Congress of two Houses, with equality of the States in the Senate; a single Executive with limited term; a three-fold distribution of the powers of government, with the judicial power vested in a Supreme Court; a delegation to Congress of enumerated powers and of others necessary and proper to carry the foregoing into effect; and finally a carefully prepared plan for amending the instrument: these and other familiar provisions commended themselves to a majority of the delegates—and at last, on the question to agree to the Constitution, all the States answered aye. Thereupon the Constitution was ordered to be engrossed and on the 17th of September the document as engrossed was read. Thereupon Franklin rose with a speech in his hand which he had reduced to writing for his own conveniency and which Mr. Wilson read in the words following:

> I confess that there are several parts of this constitution which I do not at present approve, but I am not sure I shall never approve them: For having lived long, I have experienced many instances of being obliged by better information, or fuller consideration, to change opinions even on important subjects, which I once thought right, but found to be otherwise. It is therefore that the older I grow, the more apt I am to doubt my own judgment, and to pay more respect to the judgment of others. Most men indeed as well as most sects in Religion, think themselves in possession of all truth, and that wherever others differ from them it is so far error. Steele a Protestant in a Dedication tells the Pope, that the only difference between our Churches in their opinions of the certainty of their doctrines is, the Church of Rome is infallible and the Church of England is never in the wrong. But though many private persons think almost as highly of their own infallibility as of that of their sect, few express it so naturally as a certain French lady, who in a dispute with her sister, said "I don't know how it happens, Sister but I meet with nobody but myself, that's always in the right—*Il n'y a que moi qui a toujours raison.*"

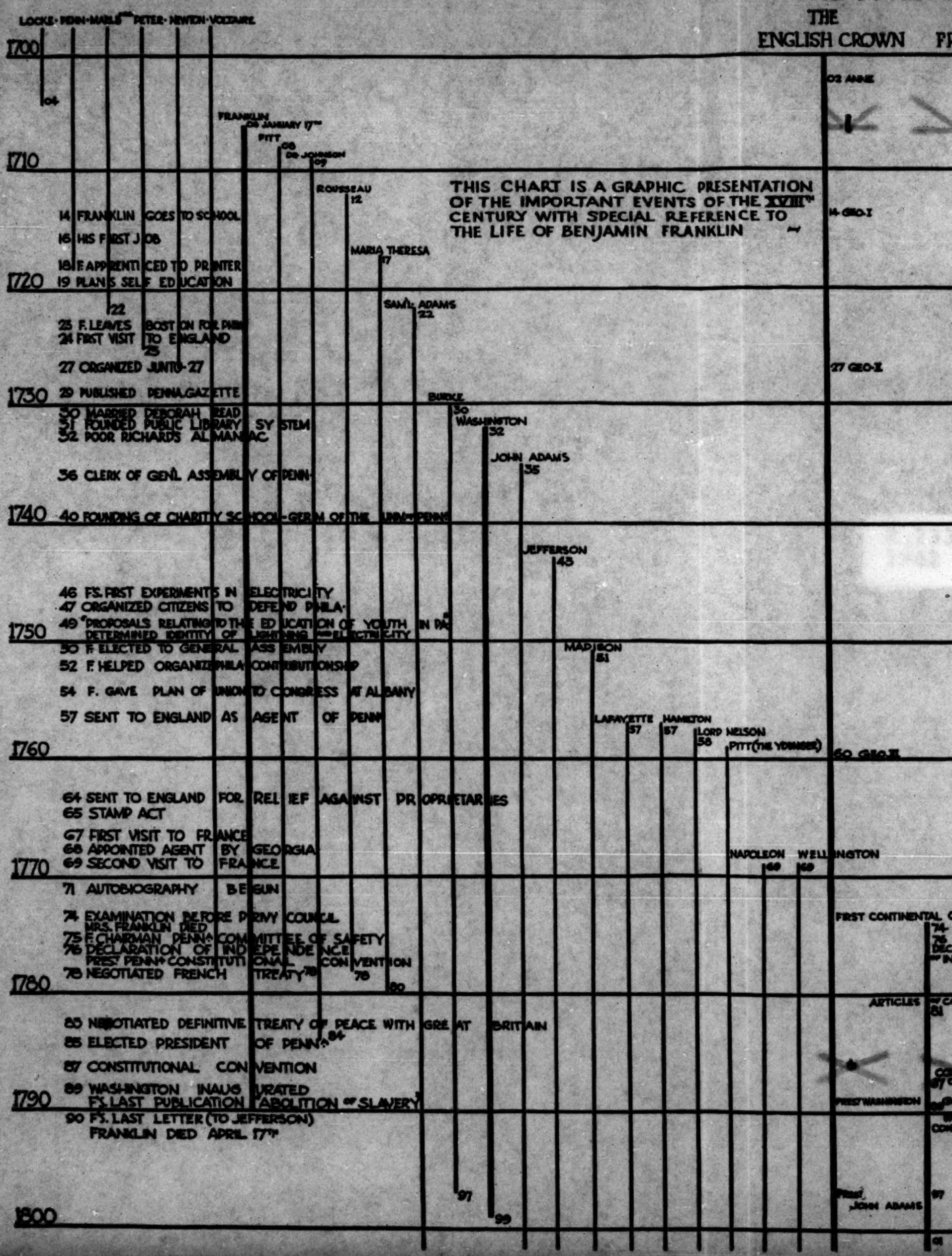
In these sentiments, Sir, I agree to this Constitution with all its faults, if they are such; because I think a general Government necessary for us, and there is no form of Government but what may be a blessing to the people if well administered, and believe farther that this is likely to be well administered for a course of years, and can only end in Despotism, as other forms have done before it, when the people shall become so corrupted as to need despotic Government, being incapable of any other. I doubt too whether any other Convention we can obtain, may be able to make a better Constitution. For when you assemble a number of men to have the advantage of their joint wisdom, you inevitably assemble with those men, all their prejudices, their passions, their errors of opinion, their local interests, and their selfish views. From such an assembly can a perfect production be expected? It therefore astonishes me, Sir, to find this system approaching so near to perfection as it does; and I think it will astonish our enemies, who are waiting with confidence to hear that our councils are confounded like those of the Builders of Babel; and that our States are on the point of separation, only to meet hereafter for the purpose of cutting one another's throats. Thus I consent, Sir, to this Constitution because I expect no better, and because I am not sure, that it is not the best. The opinions I have had of its errors, I sacrifice to the public good. I have never whispered a syllable of them abroad. Within these walls they were born, and here they shall die. If every one of us in returning to our Constituents were to report the objections he has had to it, and endeavor to gain partizans in support of them, we might prevent its being generally received, and thereby lose all the salutary effects & great advantages resulting naturally in our favor among foreign Nations as well as among ourselves, from our real or apparent unanimity. Much of the strength & efficiency of any Government in procuring and securing happiness to the people depends, on opinion, on the general opinion of the goodness of the Government, as well as of the wisdom and integrity of its Governors. I hope therefore that for our own sakes as a part of the people, and for the sake of posterity, we shall act heartily and unanimously in recommending this Constitution (if approved by Congress & confirmed by the Conventions) wherever our influence may extend, and turn our future thoughts & endeavors to the means of having it well administered.

On the whole, Sir, I can not help expressing a wish that every member of the Convention who may still have objections to it, would with me, on this occasion doubt a little of his own infallibility, and to make manifest our unanimity, put his name to this instrument.—

He then moved that the Constitution be signed by the members and offered the following as a convenient form viz. "Done in Convention by the unanimous consent of *the States* present the 17th of Sep^{r}. &c—In Witness whereof we have hereunto subscribed our names."

I note in passing Madison's observation that this "ambiguous form" as he calls it (i.e. "done in Convention by the unanimous consent of *the States* present the 17th of Sep^{r}. &c") had been drawn up by Gouverneur Morris in order to gain the

THE XVIII C



THIS CHART IS A GRAPHIC PRESENTATION OF THE IMPORTANT EVENTS OF THE XVIIITH CENTURY WITH SPECIAL REFERENCE TO THE LIFE OF BENJAMIN FRANKLIN

I CENTURY

	THE FRENCH CROWN	EVENTS IN AMERICA	EVENTS IN EUROPE
		02 QUEEN ANNE'S WAR 03 JONATHAN EDWARDS BORN 04 DEERFIELD MASSACRE 04 PUBLICATION OF THE BOSTON NEWS LETTER	1700 WAR: RUSSIA AND SWEDEN 01 THE PRUSSIAN MONARCHY 01 WAR OF THE SPANISH SUCESSION 07 KINGDOM OF GREAT BRITAIN 08 OUDENARDE: MARLBOROUGH'S THIRD VICTORY 09 MALPLAQUET: THE DECISIVE VICTORY
	15 LOUIS XV	13 PEACE OF UTRECHT	10 BERKELEY'S "PRINCIPLES" 11 CHARLES VI (FATHER OF MARIA THERESA) 13 PEACE OF UTRECHT 14 WAR: TURKS AND VENICE 15 THE PRETENDER'S REBELLION 17 TRIPLE ALLIANCE (ENGLAND, FRANCE, AUSTRIA AND LATER HOLLAND 18 DEATH OF CHARLES XII 19 ROBINSON CRUSOE 20 PRAGMATIC SANCTION
		28 BISHOP BERKELEY ARRIVED IN AMERICA 29 THE CAROLINAS DIVIDED	21 WALPOLE MINISTRY 26 GULLIVERS TRAVELS 27 WAR: ENGLAND VS AUSTRIA AND SPAIN 28 GAY'S "BEGGAR'S OPERA" 29 TREATY OF SEVILLE 29 LAW'S "SERIOUS CALL"
		32 GEORGIA COLONIZED	33 WAR OF THE POLISH SUCCESSION 33 HOGARTH: THE HARLOT'S PROGRESS THE RAKE'S PROGRESS 37 GIBBON BORN 38 PEACE OF VIENNA
		44 KING GEORGE'S WAR 48 45 CAPTURE OF LOUISBURG	40 WAR OF THE AUSTRIAN SUCCESSION 40 FREDERICK II (THE GREAT) 41 RICHARDSON'S "PAMELA" 46 CULLODEN THE YOUNG PRETENDER DEFEATED 48 PEACE OF AIX-LA-CHAPELLE 49 GOETHE BORN 49 GRAY'S ELEGY
		FRENCH AND INDIAN WAR 54-ALBANY CONGRESS 55-SEIZURE OF ACADIA 55 BRADDOCK'S DEFEAT 59 WOLFE'S CAPTURE OF QUEBEC	50 BACH DIED 54 NEWCASTLE MINISTRY 56 SEVEN YEARS WAR 56 MOZART BORN 57 CLIVE'S VICTORY AT PLASSEY 59 HANDEL DIED 60 STERNE'S TRISTRAM SHANDY
		63 JAMES KENT BORN 63 PEACE OF PARIS 65 STAMP ACT (REPEALED IN 66)	63 65 WATT'S STEAM ENGINE 65 BLACKSTONE'S COMMENTARIES 66 LORD CHATHAM'S MINISTRY 66 VICAR OF WAKEFIELD 68 ARKWRIGHT'S SPINNING MACHINE 70 LORD NORTH'S MINISTRY 70 BEETHOVEN BORN
NTAL CONGRESS 74- 76 DECLARATION OF INDEPENDENCE	74 LOUIS XVI	73 BOSTON TEA PARTY 74 BOSTON PORT BILL 75 WAR OF INDEPENDENCE 76 OCCUPATION OF NEW YORK BY BRITISH 77 OCCUPATION OF PHILA.-77 BURGOYNE'S SURRENDER VALLEY FORGE 78 FRENCH TREATY 78 HOPKINSON'S "BATTLE OF THE KEGS" EVACUATION OF PHILA.-79 JOHN PAUL JONES NAVAL VICTORY	(BEGINNING OF GREAT ENGLISH NEWSPAPERS IN THIS DECADE) 74 LORD NORTH'S FIVE ACTS 76 ADAM SMITH'S "WEALTH OF NATIONS" PRIESTLEY'S DISCOVERY OF OXYGEN 78 WAR BETWEEN GREAT BRITAIN AND FRANCE
S OF CONFEDERATI 81 87 CONSTITUTIONAL CONVENTION N 89 GOVERNMENT UNDER THE CONSTITUTION		80 ARNOLD'S TREASON 80 SLAVERY ABOLISHED IN PENNA. 81 SURRENDER OF CORNWALLIS 83 83 WASHINGTON RESIGNED HIS COMMISSION 86 TRUMBULL'S PAINTING "THE BATTLE OF BUNKER HILL" 87 SHAY'S REBELLION 88 CONGRESS APPOINTED ELECTION DAYS UNDER THE CONSTITUTION	83 PEACE OF PARIS AND VERSAILLES: MINISTRY OF PITT 84 SPOHR BORN 88 IMPEACHMENT OF WARREN HASTINGS
97 S 01	95 REIGN OF TERROR 95 DIRECTORY 99 1st CONSULATE		92 BENJ. WEST PRESIDENT OF THE ROYAL ACADEMY 93 WAR BETWEEN ENGLAND AND FRANCE 95 PARTITION OF POLAND 97 NAPOLEON CROSSED THE ALPS

dissenting members and put into the hands of Dr. Franklin that it might have the better chance of success. It failed of its purpose, however, because while the States were unanimous, 3 of the 42 members present persisted in their refusal to sign. These were Randolph, Mason and Gerry. "Whilst the last members were signing it" wrote Madison, "Doct^{r}. Franklin looking towards the Presidents Chair, at the back of which a rising sun happened to be painted, observed to a few members near him, that Painters had found it difficult to distinguish in their art a rising from a setting sun. I have said he, often and often in the course of the Session, and the vicisitudes of my hopes and fears as to its issue, looked at that behind the President without being able to tell whether it was rising or setting: But now at length I have the happiness to know that it is a rising and not a setting Sun."

It is not difficult to imagine with what unspeakable relief President Washington declared the Convention dissolved by an adjournment sine die. That which had seemed difficult at the outset, and at times during the course of the proceedings quite hopeless, had in the end been accomplished—as, I venture to believe, chiefly through the moderating influence of the only patriarch in the group. Every one of his characteristic virtues had conspired to make Franklin's influence the dominating feature of the Convention. Honesty of purpose, steadfastness in the pursuit of his objective, thoroughness of method, a decent regard for the opinions of others, a certain humility in willingness to subordinate his own and that selflessness which is patriotism at its best—all worked in harmony for the attainment of the final result.

At last the closed doors of Independence Hall were opened and the delegates as they issued from the building found themselves surrounded by a crowd of citizens eager to know what had been the outcome of the Convention's long deliberations. Among them was Mrs. Powel of Philadelphia. She approached Dr. Franklin with an anxious question: "Well, Doctor, what have we got—a Republic or a Monarchy?" "A Republic" replied the Doctor, "if you can keep it."

AN AIRPLANE VIEW OF FRANKLIN'S WORLD.

The purpose of the Chart is merely to give a bird's eye view of Franklin's world. Any such glimpse is of course scarcely more than a temptation to closer study. As to the latter half of the XVII and all of the XVIII Century reference may be made to the third and fourth volumes of Green's "A Short History of the English People." For a brilliant picture of England in particular and of Europe in general from 1650 to 1722, see Winston Spencer Churchill's "Marlborough; His Life and Times."

I. Lives.

Of the lives indicated on the Chart the six which began prior to 1700 are those of Locke, Penn, Marlborough, Peter the Great, Newton and Voltaire. Franklin, when but a lad, read Locke's famous Essay. Penn was the connecting link between the mother country and the commonwealth which was Franklin's by adoption. When the young Franklin spent a year and a half in London (1724–1726) the great Marlborough had been dead only two years. Newton was then the man most eminent in Franklin's world of science. Peter the Great is indicated merely to relate English events to the contemporary life of the Continent. Voltaire appears both because he was sojourning in London while the young Franklin was there and because the two men actually met more than half a century later.

The presence of the other indicated lives is self-explanatory except, perhaps, that of Maria Theresa. Her life-line is given because she was the most dramatic figure in the War of the Austrian Succession, the American phase of which (called King George's War) had an intimate relation to the situation with which Franklin later attempted to deal in his Albany Plan of Union.

II. Wars.

As an indication of the readiness of all European States to appeal to force, a few of the XVIII Century wars are recorded by vertical lines. If the entire war-map of Europe were to be indicated, the chart would have to show a number of other conflicts, including The Northern War (1700–1721) in which Peter the Great of Russia and Charles XII of Sweden were the principal actors; and the wars fought between the Turks and the Emperor (1716–1718). During the XVIII Century, as soon as a throne became vacant, and even in anticipation of a vacancy, there was a scramble for the place by various powers—each attempting to justify by argument a course actually dictated either by fear or greed. There were thus successive European election days—with the supporters of each candidate using bayonets instead of ballots.

---

1. The earliest of the wars indicated on the Chart is the War of the Spanish Succession (1701–1713).

The situation which gave rise to this conflict can best be understood by picturing the King of Spain (Charles II) about to die childless, while Louis XIV of France and the Hapsburg Emperor Leopold I get ready to fight one another for the Spanish Crown. As Charles is slow about dying, each of the prospective contenders gets some allies and makes an anticipatory partition (on paper) of Spain and her possessions. This irritates the childless King; but he realizes that his Crown must go somewhere—either to France or to Austria—and so he wobbles between the two groups, now encouraging one and now the other. Finally by will he bequeaths his crown to the Bourbon, Philip of Anjou, the nominee of Louis. Then Charles dies. Meanwhile England and Holland, the two great naval powers, are determined that neither France nor Austria (i.e., neither Louis XIV nor Leopold I) shall have the Spanish crown, so they make a deal with Leopold by which it is agreed that the Crown shall go not to Leopold or to his elder son but to his second son Charles —thus seeking to checkmate Louis and appease Leopold but (as it was thought) effectually preventing the union of Spain either with France or Austria. This deal of course leads to war—the Austro-English-Dutch-Prussian group lined up in support of Leopold's nominee Charles, against France and her Italian allies (Savoy and Mantua) lined up behind Louis's nominee Philip. This war, with Spain as the prize, is fought pretty much everywhere except in Spain. In Spain itself (and particularly in Castile) Philip of Anjou, Louis's nominee, is recognized as King under the title Philip V. However, Marlborough and Prince Eugene of Savoy (Savoy having switched from its French alliance to the other side) win brilliant victories over the French. The British capture Gibraltar. Leopold dies during the war; so does his elder son Joseph who reigns as emperor for only five years; so that the younger son, Charles, unexpectedly becomes emperor while still a candidate for the Spanish Crown. Meanwhile the English Whig Ministry has fallen, Marlborough's enemies have come into power, he is removed from his command; peace negotiations begin; and finally the so-called Peace of Utrecht is concluded—which is really a series of separate treaties, since it is found impossible for the various belligerents all to agree to the same terms. The Marlborough victories, which should have put candidate Charles upon the Spanish throne, actually have no such result; the throne is awarded to candidate Philip, but on condition that the Crowns of France and Spain remain forever separate; Great Britain, Holland, Savoy and Prussia get pretty much all they want (including the cession of Gibraltar and Minorca to the English and the renunciation of the Pretender's claims to the English Crown). While the nominal winner of the war, the new Emperor Charles VI, looks helplessly on, as his defeated rival (candidate Philip) gets the Spanish Crown, the prize for which the war had been fought. Largely to save his face the Emperor continues the war against France. In this he is unsuccessful; and wisely decides to conclude a treaty of peace. He, however, continues a nominal war against Spain—since a recognition of the triumph of his defeated rival, the Bourbon Philip, is too bitter a pill for the Emperor to swallow. As the curtain is rung down Louis XIV dies and the whole serio-comic performance comes to an end. Not the least tragic feature of it all has been the involvement of the American Colonies in the conflict and the shedding of blood in the Western Hemisphere as an incident of a war of European origin.

In America this War of the Spanish Succession was known as Queen Anne's War, she being on the English throne during its progress.

2. The next War between England and Spain (1718–1720) was an echo of the War of the Spanish Succession. Philip V married as his second wife Elizabeth Farnese, niece of the

Duke of Parma, whose ambitions and intrigues were extraordinary. She soon gained complete ascendency over her husband's mind. Alberoni, agent for Parma at Madrid, had suggested this marriage and in consequence of it he was given a large share in the conduct of Spanish affairs. He was thoroughly sympathetic with the Queen's Italian plans and hoped to free Italy from Austrian control. Less sympathetically he went along with her scheme to put her husband on the throne of France. Cardinal Dubois, adviser of Louis XV, checkmated the moves of Alberoni by bringing England and Holland into a triple alliance with France dedicated to the preservation of the treaties embodied in the Peace of Utrecht. To meet this formidable combination Alberoni undertook to aid and abet the Pretender in his designs on the English Crown and by an alliance with Charles XII to enlist both Sweden and Russia in the support of a Jacobite uprising against George I. This grandiose scheme came to nothing. Charles XII was killed; his minister Görz was executed; the Spanish fleet intended for the support of the Pretender was wrecked; the British fleet inflicted heavy damage on Spanish sea coast towns; and, to cap the climax, the Emperor, having gotten rid of his war with the Turks, joined the Triple Alliance —thus making it Quadruple. Alberoni was exiled and a treaty of peace made, by the terms of which Philip renounced his claim to the French Crown and the Emperor did the like in regard to the Spanish; while the ultimate succession to the Duchies of Parma and Tuscany was assured to the children of Philip and his queen. She and her husband, however, were fearful that Charles VI would disregard this latter assurance. They accordingly began a series of intrigues, first with France and then with England and finally, by the Treaty of Vienna (1725) even with Charles VI himself—whom they had begun by distrusting. War between Spain and England followed Spain's alliance with the Emperor and Gibraltar was besieged, it being a favorite project of Philip's Queen to recover for Spain Gibraltar and Minorca. Before long, however, she became convinced that her new ally Charles VI would do little to help establish her son Don Carlos in Parma and Tuscany. So she changed partners, made the Treaty of Seville (1729) with England, France and Holland and gained their support for Don Carlos by abandoning Spain's claims on Gibraltar and Minorca. A move hostile to Don Carlos by Charles VI was met by the diplomacy of Walpole for England and Fleury for France, who induced the Emperor to confirm by the second Treaty of Vienna his assurance respecting Don Carlos by a promise that England would guaranty the Emperor's favorite project, the Pragmatic Sanction, by which (in default of male issue) his daughter Maria Theresa was to be assured of the succession to all lands united under the Austrian sceptre.

3. The next war indicated on the Chart, the War of the Polish Succession (1733–1738), was the lineal successor of the Spanish War last above described. Following the second Treaty of Vienna Philip's Queen Elizabeth continued her intrigues and adroitly succeeded in accomplishing what the War of the Spanish Succession was fought to prevent—an alliance (1733) between the French and Spanish branches of the House of Bourbon. These allies now stood ready to oppose England and the Austrian Emperor in order to destroy England's commercial monopoly and to drive the Hapsburgs out of Italy. An opportunity to go into action came when in 1733 the Polish throne became vacant through the death of Augustus II. Instantly two candidates threw their hats into the ring: Stanislaus Lesczinski, father-in-law of Louis XV, and Augustus III, Elector of Saxony. The former was backed by the new French-Spanish combination and the latter by a partnership between Austria and Russia. War immediately followed. France and Spain were successful from the start. Their victory might have been expected to place their candidate Lesczinski on the Polish throne. By the Peace of Vienna (1738) however, which took three years to negotiate after hostilities were suspended, he renounced his claim and was given instead a life-interest in the Duchies of Lorraine and Bar. Austria ceded Naples, Sicily and Elba to Spain. France was awarded a remainder-interest in Lorraine and Bar and agreed to guaranty the Pragmatic Sanction. Thus another war ended in a way wholly different from that which the victors had planned. France and Spain won the war; but their candidate did not get the Polish throne.

4. In 1740 the time arrived for testing the value of the

various guaranties of the Pragmatic Sanction. The Emperor Charles VI died without male issue and, according to the promises made by everybody at her father's urgent request, his daughter Maria Theresa became entitled to succeed to all the Austrian dominion, including Poland. Her succession, which had been so carefully planned by her father, immediately became the subject of a new war—the War of the Austrian Succession. All the guarantors except England declined to honor their commitments and it looked as if Maria Theresa's cause was hopeless. Her courage, however, was great and she took quick advantage of various unforeseen developments. The candidates for the imperial throne (other than herself) were Charles Albert of Bavaria who never had signed the Pragmatic Sanction and two candidates who had approved it—Philip V of Spain and Augustus III of Saxony. These two were now quite ready to dishonor their commitments. Frederick the Great, though not a candidate for the imperial throne, advanced a claim to take half of Silesia away from Austria and nourished a determination to take it all. France, Spain, Saxony and Prussia made common cause with Bavaria and supported Charles Albert who was proclaimed Emperor in 1742 as Charles VII. Maria Theresa, supported by Austria and Hungary, concluded an alliance with England. Following military victories by Frederick, Maria Theresa ceded him the territory which he coveted and he withdrew from the alliance against her. She then proceeded to defeat the remaining allies of her rival Charles Albert, whereupon Frederick, fearing that the Queen's success might tempt her to revoke the Silesian cession, proceeded to make a new deal with Charles Albert and the second stage of the war began. Frederick's alliance with Charles Albert provoked a counter-alliance against Prussia by Austria, Saxony, England and Holland. Charles Albert at this point opportunely died; and Maria Theresa concluded a separate peace with the new Elector of Bavaria by which lands taken from Bavaria were restored and the vote of Bavaria at the Imperial Election was pledged to Maria Theresa's husband, Francis Stephen of Lorraine. The pledge was kept and in 1745 Francis Stephen was duly elected Emperor and ascended the throne as Francis I. Frederick recognized him as such in return for a confirmation of the Silesian cession. The Peace of Aix-la-Chapelle (1748) embodied these terms and also assured the House of Hanover in its right of succession both in its German possessions and in Great Britain. As will presently appear, however, the treaty was far from being a settlement, because Austria was sullen over the concessions made to Frederick and the terms of the treaty proved to be the breeder of another war.

5. The Seven Years' War (1755 to 1763) was fought in America as well as in Europe. In America it is usually styled the French and Indian War and was actually begun there in 1755 between France and England without any formal declaration of hostilities. In Europe the course of events, following Aix-la-Chapelle, had been influenced by the desire of Maria Theresa to get back Silesia, by the anxiety of George II of England to be secure in his possession of Hanover and by the policy of France to isolate England from continental support. Maria Theresa incurred afresh the hostility of Frederick by an alliance with his personal enemy, Elizabeth, Empress of Russia (1746). France and Austria were drawn closely together for common defense against Prussia; while George II, fearing Frederick as a threat to Hanover, made in 1756 the Westminster Treaty of Neutrality with him. This aroused indignation in France and was promptly followed by a formal offensive alliance between France and Austria. In June of 1756 hostilities in Europe broke out between France and England and almost simultaneously Frederick attacked the French and Austrian allies who were later reinforced by Russia. As between Prussia and this continental group the war was bitterly fought during five years. Frederick, supported by military subsidies from George II, won many brilliant but costly victories. The accession of George III in 1760 stopped the subsidies; but Frederick was saved from collapse by the death of his arch-enemy, Elizabeth of Russia. This caused Russia to withdraw from the alliance against him. Meanwhile the conflict between France and England had been proceeding in America and in 1762–1763 a separate peace was made between those two powers and Spain styled the Peace of Paris. This in turn induced a treaty (Hubertusburg) between Austria and Prussia; and another bloody and indecisive war thus came to an end.

It is impossible to read the story of these wars without realizing two things: (1) how true was Washington's observation that European controversies have causes essentially foreign to our concerns; and (2) how ineffective are the vast majority of wars when war is considered as an instrument for settling international controversies.

"It is our true policy" Washington said "to steer clear of permanent alliances with any portion of the foreign world." In a recent message President Roosevelt attributed to him the expression "*entangling* alliances." This was *not* Washington's phrase. It will be found in Jefferson's First Inaugural.

III. Events in America and Europe.

The course of descent of the Crown in England and France and the graphic presentation of the evolution of the government of the United States supply a sort of background to the picture.

---

1. The number of recorded events in America might of course be readily increased. An interested student could profitably make a Chart of his own and amplify the recorded items indefinitely.

2. The same thing is true of events in Europe. Those mentioned on the Chart are merely suggestive of the contrasts in which the XVIII Century abounded—bloody wars, a vital literature and notable contributions to the arts and sciences. Later in the century Franklin was to meet with great men on equal terms. But of the period when he visited London as a young man Van Doren has this to say: "It was a brilliant age—and Franklin was little closer to it at first hand than if he had been in Boston or Philadelphia. He read and heard of great men, barely saw or talked with them."

SOME IMPROVEMENTS IN PRACTICAL FOURIER ANALYSIS AND THEIR APPLICATION TO X-RAY SCATTERING FROM LIQUIDS.

BY

G. C. DANIELSON AND C. LANCZOS,

Purdue University, Lafayette, Indiana.

(*Continued from April issue.*)

4. Testing the Convergence of the Fourier Series.

As mentioned in the troduction, the principal difficulty of trigonometric interpolation is the uncertainty regarding the number of ordinates necessary for sufficient accuracy. Since we cannot judge by inspection the number of ordinates necessary and since the vanishing of the last two or three coefficients gives no absolute guarantee of the convergence, it seems desirable to test the probable convergence of the Fourier series by some preliminary calculations before we start our analysis. The two simple tests we shall outline test the convergence of the sine and the cosine series separately. The two tests are independent of each other so that, if both indicate sufficient convergence, we can assume that the number of coefficients is probably large enough. While it is not possible to guarantee the absolute convergence, if either test does not indicate sufficient convergence we can be sure the number of coefficients is not sufficiently great.

The alternate sum test is based on the value of the nth coefficient in an analysis for $2n$ coefficients.

In equations (12) and (13) let $k = n$. Then

$$a_n = \frac{1}{n}(u_0 - u_2 + u_4 - u_6 + \cdots u_{2n}), \qquad (61)$$

$$b_n = \frac{1}{n}(v_1 - v_3 + v_5 - v_7 + \cdots v_{2n-1}). \qquad (62)$$

If we apply these relations to the double number of ordinates we obtain a_{2n} and b_{2n} which must each be negligible if both the cosine analysis for $2n$ coefficients and the sine analysis for $2n$ coefficients are to give sufficient accuracy.

The above calculation tests only one coefficient (viz. the last one). This test has a purely orienting value: that is, if this coefficient is too large then it is certain that one has to use a larger number of harmonics. If, however, this coefficient vanishes, it is still possible that there are inaccuracies in the calculated harmonics owing to higher terms which have been neglected. A more comprehensive and dependable accuracy guide is obtained by taking the alternate sum of the squares for the original number of ordinates and comparing this with the same sum for the double number of ordinates. If these are practically equal, the existence of appreciable higher harmonics is very improbable.

The alternate square sum test is based upon the following well-known relations for any orthgonal set of functions

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)dx = \tfrac{1}{2}a_0^2 + a_1^2 + a_2^2 + \cdots \\ + b_1^2 + b_2^2 + \cdots \qquad (63)$$

and

$$\tfrac{1}{2}a_0^2 + \sum_{k=1}^{2n-1} a_k^2 = \frac{u_0^2 + u_1^2 + \cdots u_{2n}^2}{2n},$$

$$\sum_{k=1}^{2n} b_k^2 = \frac{v_1^2 + v_2^2 + \cdots v^2_{2n-1}}{2n}. \qquad (64)$$

If we divide the function $f^2(x)$ into its odd component $f_1^2(x)$ and its even component $f_2^2(x)$ then

$$I_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} f_1^2(x)dx = \tfrac{1}{2}a_0^2 + \sum_{k=1}^{2n-1} a_k^2,$$

$$I_2 = \frac{1}{\pi}\int_{-\pi}^{\pi} f_2^2(x)dx = \sum_{k=1}^{2n} b_k^2. \qquad (65)$$

From equations (64) and (65) it follows that

$$I_1 = \frac{u_0^2 + u_1^2 + \cdots u_{2n}^2}{2n},$$

$$I_2 = \frac{v_1^2 + v_2^2 + \cdots v^2_{2n-1}}{2n}. \qquad (66)$$

In these expressions the values for u and v are determined by equation (7) and can, therefore, be obtained directly from the original curve. Now, instead of producing the definite integrals I_1 and I_2 by plotting the $f_1^2(x)$ and $f_2^2(x)$ curves and measuring the areas by a planimeter, we can obtain them in an indirect way by applying equations (66) again but now to twice the original number of ordinates. Since I_1 and I_2 are independent of the number of readings, a comparison of this sum to the original sum gives a good indication of the validity of our approximation.

Let

$$s_1 = \frac{1}{2n}(u_0^2 + u_2^2 + u_4^2 + \cdots u_{4n}^2) \tag{67}$$

and

$$s_1' = \frac{1}{4n}(u_0^2 + u_1^2 + u_2^2 + u_3^2 + u_4^2 + \cdots u_{4n}^2). \tag{68}$$

If the convergence is sufficiently good for the cosine analysis, $s_1' - s_1$ is negligible. That is,

$$u_0^2 - u_1^2 + u_2^2 - u_3^2 + \cdots u_{4n}^2 \tag{69}$$

is negligible.

The percentage error δ_1 is

$$\frac{s_1' - s_1}{s_1'} \times 100$$

$$= \frac{(u_0^2+u_1^2+u_2^2+\cdots) - 2(u_0^2+u_2^2+u_4^2+\cdots)}{(u_0^2+u_1^2+u_2^2+\cdots)} \times 100, \tag{70}$$

$$\delta_1 = \frac{(u_1^2+u_3^2+u_5^2+\cdots) - (u_0^2+u_2^2+u_4^2+\cdots u_{4n}^2)}{(u_1^2+u_3^2+u_5^2+\cdots) + (u_0^2+u_2^2+u_4^2+\cdots u_{4n}^2)} \times 100. \tag{71}$$

Similarly let

$$s_2 = \frac{1}{2n}(v_2^2 + v_4^2 + \cdots v_{4n}^2), \tag{72}$$

$$s_2' = \frac{1}{4n}(v_1^2 + v_2^2 + v_3^2 + \cdots v_{4n}^2). \tag{73}$$

If the convergence is sufficiently good for the sine analysis, $s_2' - s_2$ is negligible. That is

$$v_1^2 - v_2^2 + v_3^2 - v_4^2 + \cdots v_{4n}^2 \tag{74}$$

is negligible.

The percentage error δ_2 is

$$\frac{s_2' - s_2}{s_2'} \times 100 = \frac{(v_1^2 + v_2^2 + v_3^2 + \cdots) - 2(v_2^2 + v_4^2 + \cdots)}{(v_1^2 + v_2^2 + v_3^2 + \cdots)} \times 100, \quad (75)$$

$$\delta_2 = \frac{(v_1^2 + v_3^2 + v_5^2 + \cdots v_4{}^2{}_{n-1}) - (v_2^2 + v_4^2 + v_6^2 + \cdots v_{4n}^2)}{(v_1^2 + v_3^2 + v_5^2 + \cdots v_4{}^2{}_{n-1}) + (v_2^2 + v_4^2 + v_6^2 + \cdots v_{4n}^2)} \times 100. \quad (76)$$

δ_1 and δ_2 represent twice the percentage error in the reading of an original ordinate.

An analysis has been carried out with 8, 16, 32, and 64 harmonics. Using these tests the deviation is about 30 per cent. for the first two sets indicating that higher harmonics have to be used. However, 32 harmonics reduced the deviation to 2.5 per cent. and 64 to 0.5 per cent. Depending upon the nature of the investigation one selects an accuracy which is comparable to but better than the experimental error inherent in the problem.

This test is superior to the alternate sum test which involves only one harmonic. However, the test is still only a necessary and not a sufficient condition for convergence of the Fourier series.

5. Evaluation of the Fourier Integrals (or Interpolation for Intermediate Points).

The Fourier integrals given in equations (2) and (3) for a continuous variable *t* are

$$\varphi_1(t) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos tx \, dx \quad (77)$$

and

$$\varphi_2(t) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin tx \, dx. \quad (78)$$

These may be written as

$$\varphi_1(t) = \frac{1}{\pi} \int_{0}^{\pi} [f(x) + f(-x)] \cos tx \, dx \quad (79)$$

and

$$\varphi_2(t) = \frac{1}{\pi} \int_{0}^{\pi} [f(x) - f(-x)] \sin tx \, dx. \quad (80)$$

Let the function $f(x)$ be divided into its even component $f_1(x)$ and its odd component $f_2(x)$. Then

$$f(x) = f_1(x) + f_2(x) \tag{81}$$

and

$$f(-x) = f_1(x) - f_2(x). \tag{82}$$

Hence

$$2f_1(x) = f(x) + f(-x) \tag{83}$$

and

$$2f_2(x) = f(x) - f(-x). \tag{84}$$

Equations (79) and (80) can now be written as

$$\varphi_1(t) = \frac{2}{\pi}\int_0^\pi f_1(x)\cos tx\,dx \tag{85}$$

and

$$\varphi_2(t) = \frac{2}{\pi}\int_0^\pi f_2(x)\sin tx\,dx. \tag{86}$$

To illustrate the method of interpolation we shall consider the equation (86) only, but the same procedure applies to equation (85). Dropping the subscripts equation (86) becomes

$$\varphi(t) = \frac{2}{\pi}\int_0^\pi f(x)\sin tx\,dx. \tag{87}$$

Since $f(x)$ is an odd function, we may represent it as a sum of sine terms

$$f(x) = \sum_{k=1}^{2n} b_k \sin kx, \tag{88}$$

where all b_k are known from a Fourier analysis.

$$b_k = \frac{1}{n}\sum_{\alpha=1}^{2n-1} v_\alpha \sin\frac{\pi}{2n}k\alpha. \tag{89}$$

Substituting (89) in (87)

$$\varphi(t) = \frac{2}{\pi}\int_0^\pi \sum_{k=1}^{2n} b_k \sin kx \sin tx\,dx, \tag{90}$$

$$\varphi(t) = \frac{2}{\pi}\sum_{k=1}^{2n} b_k \int_0^\pi \sin kx \sin tx\,dx. \tag{91}$$

The integral is easily evaluated by expansion into a difference of two cosines.

$$\varphi(t) = \frac{2}{\pi}\sum_{k=1}^{2n} b_k \frac{1}{2}\left\{\frac{\sin (k - t)\pi}{k - t} - \frac{\sin (k + t)\pi}{k + t}\right\}. \tag{92}$$

Since k is integral,

$$\sin (k - t)\pi = -(-1)^k \sin t \tag{93}$$

and

$$\sin (k + t)\pi = (-1)^k \sin t. \tag{94}$$

The final formula[7] is, therefore,

$$\varphi(t) = \frac{\sin t\pi}{\pi}\sum_{k=1}^{2n} b_k \left\{\frac{1}{t - k} - \frac{1}{t + k}\right\}(-1)^k. \tag{95}$$

To show that the reciprocals in equation (95) need be evaluated for only one interval let

$$t = t_0 + h \qquad \text{where} \qquad |h| \leqq 0.5. \tag{96}$$

That is t_0 is the integer which is nearest to t.

Also let

$$k' = t_0 + k \qquad \text{and} \qquad k'' = t_0 - k. \tag{97}$$

Then (95) becomes

$$\varphi(t) = \frac{\sin h\pi}{\pi}\sum_{k=1}^{2n} b_k \left\{\frac{1}{h + k''} - \frac{1}{h + k'}\right\}(-1)^{k'}, \tag{98}$$

where k' and k'' are integers.

By writing down five reciprocal tables ($h = 0.1, 0.2, 0.3, 0.4$ and 0.5) we can select the proper reciprocals to interpolate for as many as 9 intermediate values of our integral between any two consecutive Fourier coefficients.

The only disadvantage of equation (95) or (98) is the slow convergence of the reciprocals. This means a rather large

---

[7] This formula may be checked by placing $t = m$ where m is integral. Since $\sin m\pi = 0$, $\varphi(m) = 0$ for every integral value of m except for the indeterminate case $m = k$. Let $m = k + \epsilon$ where ϵ is infinitesimal.

$$\begin{aligned} \varphi(m) &= \left\{\frac{\sin (k + \epsilon)\pi}{\pi}\right\} b_{m-\epsilon}\frac{1}{\epsilon}(-1)^k \\ &= \frac{1}{\epsilon} b_{m-\epsilon}(-1)^k \left\{(-1)^k \frac{\epsilon\pi}{\pi}\right\}. \\ &= b_m \quad \text{as} \quad \epsilon \to 0. \end{aligned}$$

number of terms (say 20) may be required for the determination of a single point. This convergence may be improved by a transformation of the b_k Fourier coefficients. Let these transformed coefficients $\bar{b}_k$ be defined by the equations

$$\bar{b}_k + \bar{b}_{k+1} = b_k \tag{99}$$

and

$$\bar{b}_{2n+1} = 0. \tag{100}$$

The transformation is then obtained by the following scheme (see equation 55)

$$\begin{array}{lcccccccc}
 & b_1 & b_2 & b_3 & \dots & & b_{2n-1} & b_{2n} & \\
 & \bar{b}_2 \nwarrow & \bar{b}_3 \nwarrow & \bar{b}_4 \nwarrow & \dots & \bar{b}_{2n-1} \nwarrow & \bar{b}_{2n} \nwarrow & 0 \nwarrow & \\
\hline
 & \searrow & \searrow & \searrow & & & \searrow & \searrow & \searrow \\
\text{Differences} & \bar{b}_1 & \bar{b}_2 & \bar{b}_3 & \dots & & \bar{b}_{2n-1} & \bar{b}_{2n} & 0
\end{array}$$

The new interpolation formula is very similar to the original formula (95)

$$\varphi(t) = \frac{\sin t\pi}{\pi} \sum_{k=1}^{2n} \bar{b}_k \left[\frac{1}{(t-k)(t-k+1)} + \frac{1}{(t+k)(t+k-1)} \right] (-1)^k. \tag{101}$$

This new formula has a much better convergence since the factor by which $\bar{b}_k$ is multiplied decreases much more rapidly.

APPLICATION OF IMPROVEMENTS TO X-RAY SCATTERING FROM LIQUIDS.

1. Transformation of a Fourier Integral into a Sum.

According to the theory of Zernike and Prins [1] the density distribution in a liquid is given by

$$4\pi r^2 \rho(r) = 4\pi r^2 \rho_0 + \frac{2r}{\pi} \int_0^\infty s i(s) \sin sr\, ds, \tag{102}$$

where r is the distance measured from any particular atom, $s = (4\pi \sin\theta)/\lambda$, $\rho(r)$ is the density function, ρ_0 is the average density of the sample, and

$$i(s) = \frac{I_{\text{exp}} - I_{\text{inc}} - I_{\text{coh}}}{I_{\text{coh}}}. \tag{103}$$

It is assumed that $si(s) = 0$ from a finite value $r = a$ to $r = \infty$.

The integral term $I(r)$ can be written in a more convenient form by introducing a new independent variable

$$x = \frac{\pi}{a} s, \tag{104}$$

letting

$$f(x) = si(s) \tag{105}$$

and making a linear change in scale of the parameter r by using

$$t = \frac{a}{\pi} r. \tag{106}$$

Then

$$I(t) = t\varphi(t), \tag{107}$$

where

$$\varphi(t) = \frac{2}{\pi} \int_0^\pi f(x) \sin tx \, dx. \tag{108}$$

Since $f(x)$ is given only in the half range 0 to π, we may define $f(-x) = -f(x)$ in the range 0 to $-\pi$. This makes $f(x)$ an odd function so that it can be represented by a sum of sine terms only.

Let

$$f(x) = \sum_{k=1}^{2n} b_k \sin kx, \tag{109}$$

where k is an integer and $2n$ is an even number sufficiently great so that the sum of higher terms is negligible.

If y_m denotes the known values of the function $f(x)$ at equidistant intervals $x_m = m(\pi/2n)$, the orthogonality property of the Fourier sum gives

$$\begin{aligned} \sum_{m=1}^{2n} y_m \sin kx_m &= 0 \quad \text{if} \quad k \neq m \\ &= \frac{2n}{2} b_k \quad \text{if} \quad k = m. \end{aligned} \tag{110}$$

Hence, the Fourier coefficients are

$$b_k = \frac{1}{n} \sum_{m=1}^{2n} y_m \sin kx_m, \tag{111}$$

and $\varphi_k = b_k$ is obtained at equidistant intervals by evaluating this sum.

The desired integral

$$I(k) = kb_k = \frac{k}{n} B_k \quad (k = 1, 2, \cdots 2n), \tag{112}$$

where

$$B_k = \sum_{m=1}^{2n} y_m \sin m \frac{k\pi}{2n} \cdot \tag{113}$$

2. The Analysis of Lithium Chloride.

233
12

To illustrate the methods used in evaluating this sum we shall consider the analysis of molten lithium chloride which was analyzed previously by Lark-Horovitz and Miller.[8] The equidistant ordinates are read directly off the $si(s)$ curve calculated from the microphotometer trace of their diffraction pattern. (See Fig. 1.)

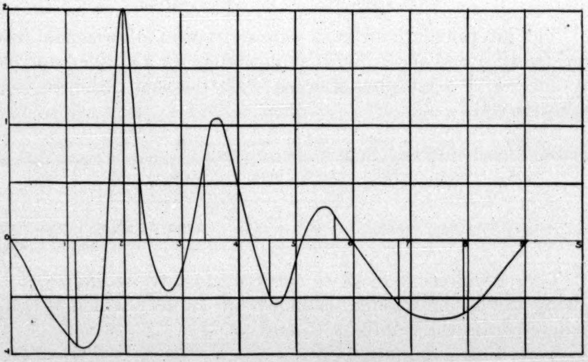


FIG. 1. $si(s)$ Curve for molten LiCl.

---

[8] Lark-Horovitz and Miller, Phys. Rev. **47**, 813 (1935).

We begin by making a sine analysis for any arbitrary small, even number of coefficients (let us say $2n = 8$) and then use the doubling process to increase this number until the convergence is sufficiently good. In this way a Fourier analysis for 16 coefficients follows from two, 32 from four, and 64 from eight analyses for 8 coefficients.

Sine Analysis for 8 Coefficients.

Figure 1 shows the $si(s)$ curve for molten LiCl. We read the 8 ordinates which are indicated by vertical lines. After multiplying by 1000 to avoid decimals the original ordinates are

$$-920,\ 605,\ 665,\ -490,\ 235,\ -565,\ -650. \tag{114}$$

The complete analysis according to the sine scheme is shown in Table 1 which is constructed from the relations 17 and 43.

The Fourier coefficients $b_k = B_k/4$ do not yet show any convergence so a further analysis is necessary. These values of B_k are B_k'' of the analysis for 16 coefficients. We now proceed with an analysis for 16 coefficients by one additional analysis for 8 coefficients.

Sine Analysis for 16 Coefficients.

The intermediate ordinates are written in a horizontal row and transformed according to equation 54 for a cosine analysis by successive additions, starting with the last ordinate (see equation 55).

$$\begin{array}{lccccccccc}
v_{2k-1} & -530 & 140 & -415 & 560 & -20 & -190 & -690 & -350 & \\
 & -965 & -1105 & -690 & -1250 & -1230 & -1040 & -350 & 0 & \\
 & \nwarrow & \nwarrow & \nwarrow & \nwarrow & \nwarrow & \nwarrow & \nwarrow & \nwarrow & \\
\hline
 & \searrow & \searrow & \searrow & \searrow & \searrow & \searrow & \searrow & \searrow & \\
\bar{V}_k & -1495 & -965 & -1105 & -690 & -1250 & -1230 & -1040 & -350 & 0
\end{array} \tag{115}$$

The complete analysis of the transformed ordinates, following the cosine scheme, is shown in Table 2 which is constructed from the relations 16 and 37.

According to equation 58

$$B_k' = A_k\left(2 \sin k\frac{\pi}{16}\right). \tag{116}$$

TABLE I.

Sine Analysis for the 8 Original Ordinates.

v_α	1 -920	2 605	3 665	4 -490
	7 -650	6 -565	5 235	
v_α'(sum)	1 -1570	2 40	3 900	4 -490
v_α''(diff)	1 -270	2 1170	3 430	——

$1 = 0.38268 = \sin \frac{\pi}{8}$ $0 = 0 = \sin 0 \frac{\pi}{8}$

$3 = 0.92388 = \sin 3 \frac{\pi}{8}$ $2 = 0.70711 = \sin 2 \frac{\pi}{8}$

$4 = 1.0000 = \sin 4 \frac{\pi}{8}$

k	$B_k = 4b_k$
1	-232
2	1283
3	-1276
4	-700
5	-2312
6	-1057
7	692
8	0

v_1' -1570	v_3' 900	——	v_2' 40	v_4' -490	——
1 -601	3 831	B_1' 230	2 28	4 -490	B_1'' -462
3 -1450	-1 -344	B_3' -1794	2 28	-4 490	B_3'' 518
v_1'' -270	v_3'' 430	——	v_2'' 1170	——	
2 -191	2 304	B_2' 113	4 1170	B_2'' 1170	
4 -270	-4 -430	B_4' -700			

Numbers in upper left hand corner represent sine values which are to be multiplied by the value of v' or v'' at the top of the column. B_k' and B_k'' are row sums.

$$B_k = B_k' + B_k'', \qquad B_{8-k} = B_k' - B_k'', \qquad b_k = \frac{B_k}{4}.$$

TABLE 2.

Cosine Analysis of the 9 Transformed Ordinates.

u_α	0 −1495	1 −965	2 −1105	3 −690	4 −1250
	8 0	7 −350	6 −1040	5 −1230	——
u_α''(sum)	0 −1495	1 −1315	2 −2145	3 −1920	4 −1250
u_α'(diff)	0 −1495	1 −615	2 −65	3 540	——

$1 = 0.92388 = \cos\frac{\pi}{8}$

$3 = 0.38268 = \cos 3\frac{\pi}{8}$

$0 = 1 = \cos 0\frac{\pi}{8}$

$2 = 0.70711 = \cos 2\frac{\pi}{8}$

$4 = 0 = \cos 4\frac{\pi}{8}$

k	$A_k = 4a_k$
0	−7378
1	−1153
2	930
3	−1436
4	147
5	32
6	74
7	−431
8	−908

u_1' −615	u_3' 540	——	u_0' −748	u_2' −65	——	
1 −568	3 207	A_1' −381	0 −748	2 −46	A_1'' −792	
3 −235	−1 −499	A_3' −734	0 −748	−2 46	A_3'' −702	
u_1'' −1315	u_3'' −1920	A_0' −3235	u_0'' −748	u_2'' −2145	u_4'' −1250	A_0'' −4143
2 −930	−2 1358	A_2' 428	0 −748	4 0	−0 1230	A_2'' 502
			0 −748	−0 2145	0 −1250	A_4'' 147

Numbers in upper left hand corner represent cosine values which are to be multiplied by the value of u' or u'' at the top of the column. A_k' and A_k'' are row sums.

$$A_k = A_k'' + A_k', \qquad A_{8-k} = A_k'' - A_k'', \qquad a_k = \tfrac{1}{4}A_k.$$

The final coefficients are given by the relations

$$B_k = B_k' + B_k'', \quad B_{16-k} = B_k' - B_k'' \quad (k-0, 1, 2 \cdots 8). \tag{117}$$

The 16 coefficients are given in Table 3.

Sine Analysis for 32 Coefficients.

Since the convergence of the coefficients is still not sufficiently good we proceed with an analysis for 32 coefficients by making one additional analysis for 16 coefficients. According to equation 48, the values of B_k just obtained are the B_k'' of an analysis for 32 coefficients.

TABLE 3.

Final Calculations of the Sine Analysis for 16 Coefficients.

k.	$2 \sin k \frac{\pi}{16}$.	B_k'.	B_k''.	B_k.	B_{16-k}.
0	0	0	0	0	0
1	.3902	−450	−232	−682	−218
2	.7654	712	1283	1995	−371
3	1.1111	−1596	−1276	−2872	−320
4	1.4142	208	−700	−492	908
5	1.6629	53	−2312	−2259	2365
6	1.8478	137	−1057	−920	1194
7	1.9616	−845	692	−153	−1537

The 16 intermediate ordinates are transformed for a cosine analysis as in equation 115.[9]

$$\begin{array}{rrrrrrrrr}
v_{2k-1} & & -230 & -745 & -905 & 2010 & -290 & -20 & 1070 \\
\bar{v}_k & \underline{-1765} & -1515 & \underline{-770} & 135 & \underline{-1875} & -1585 & \underline{-1565} & -2635 \\
-40 & -470 & 260 & 25 & -380 & -660 & -685 & -525 & -160 \\
\underline{-2395} & -2125 & \underline{-2385} & -2410 & \underline{-2030} & -1370 & \underline{-685} & -160 & 0
\end{array} \tag{118}$$

The cosine analysis of these transformed ordinates represents, after multiplication by the weight factor $2 \sin k(\pi/32)$, the contribution of the odd ordinates to the 32 coefficient analysis (see equation 58). That is,

---

[9] The second row is identical to the third and can, therefore, be omitted. After addition the sum is inserted on the left rather than below the numbers.

$$B_k' = A_k\, 2 \sin k \frac{\pi}{32} \quad (k = 0, 1, 2 \cdots 16). \tag{119}$$

To obtain these A_k we can apply the reduction process again. The even u_k (underlined) are analyzed directly by the cosine scheme for 8 coefficients and represent the A_k'' of the analysis for 16 coefficients. The odd u_k are transformed by successive subtractions according to equation 55.

$$\begin{array}{cccccccccc}
u_k & -1515 & 135 & -1585 & -2635 & -2125 & -2410 & -1370 & -160 & \\
 & -10 & -125 & 1710 & 925 & 1200 & 1210 & 160 & 0 & \\
 & \nwarrow & \nwarrow & \nwarrow & \nwarrow & \nwarrow & \nwarrow & \nwarrow & \nwarrow & \\
\bar{u} & -1525 & 10 & 125 & -1710 & -925 & -1200 & -1210 & -160 & 0
\end{array} \tag{120}$$

The cosine analysis of these 9 transformed ordinates represents, after multiplication by the weight factor $2 \cos k(\pi/16)$ (see equation 57), the A_k' of the 16 coefficient analysis. Then

$$A_k = A_k'' + A_k', \quad A_{16-k} = A_k'' - A_k' \quad (k = 0, 1, \cdots 8). \tag{121}$$

The calculation of A_k is shown in Table 4.

TABLE 4.

The Calculation of A_k for the 17 Transformed Ordinates.

k.	A_k''.	A_k^*.	A_k'.	A_k.	A_{16-k}.
0	−12787	−5832	−11664	−24451	−1123
1	−537	144	282	−255	−819
2	3477	2115	3908	−7385	−431
3	−1783	−1170	−1946	−3729	163
4	428	−602	−851	−423	1279
5	−201	−2242	−2491	−2692	2290
6	−51	−1789	−1369	−1420	1318
7	−1007	220	86	−921	−1093
8	−1077	288	0	−1977	

A_k'' are from the analysis of the even u_k

A_k^* are from the analysis of the transformed u_k

$$A_k' = A_k^*\, 2 \cos k \frac{\pi}{16}; \qquad A_k = A_k'' - A_k', \qquad A_{16-k} = A_k'' - A_k'.$$

Substitution of these values for A_k in equation 119 gives B_k'. The 32 coefficients are then

$$B_k = B_k' + B_k'', \quad B_{32-k} = B_k' - B_k'' \quad (k = 0, 1, \cdots 16). \tag{122}$$

The final calculation of the 32 coefficients is shown in Table 5.

From equation (112) the desired integral is

$$I(k) = \frac{k}{16} B_k \quad (k = 1, 2, \cdots 32). \tag{123}$$

These values were calculated and the transform curve plotted on a flat background (Fig. 2) and on the parabolic background 4 (Fig. 3). The abscissæ r are given by equation 106 ($r = (\pi/a)k$).

Testing the Convergence of the Analysis.

Since the last few coefficients B_{29}, B_{30}, and B_{31} hardly seem negligible we might suspect that our convergence is

TABLE 5.

Final Calculation of the Sine Analysis for 32 Coefficients.

k.	A_k.	$2 \sin \frac{\pi}{32} k$.	B_k'.	B_k''.	B_k.	B_{32-k}.
0	−24451	0	0	0	0	0
1	−255	.1960	−50	−682	−732	632
2	7385	.3902	2681	1995	4876	886
3	−3729	.5806	−2165	−2872	−5037	707
4	−423	.7654	−324	−492	−816	168
5	−2692	.9428	−2538	−2259	−4797	−279
6	−1420	1.1111	−1578	−920	−2498	−658
7	−921	1.2688	−1169	−153	−1322	−1016
8	−1977	1.4142	−2796	−1816	−4612	−980
9	−1093	1.5460	−1690	−1537	−3227	−153
10	1318	1.6629	2192	1194	3386	998
11	2290	1.7638	4039	2365	6404	1674
12	1279	1.8478	2363	908	3271	1455
13	163	1.9139	312	−320	−8	632
14	−431	1.9616	−845	−571	−1416	−274
15	−819	1.9904	−1630	−218	−1848	−1412
16	−1123	2.0000	−2246	0	−2246	

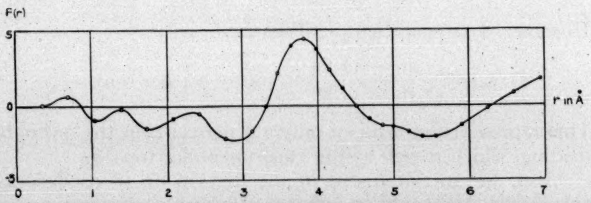


FIG. 2. Atomic density integral for molten LiCl.

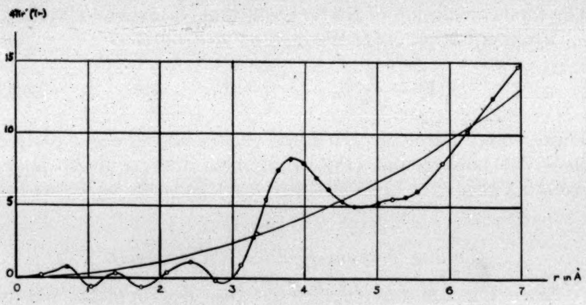


FIG. 3. Atomic density distribution for molten LiCl.

still not sufficiently good. To investigate this, the alternate square sum test was applied. Applying equation (72) to the 8 original ordinates

$$\begin{aligned} s_2 &= \tfrac{1}{8}(v_2^2 + v_4^2 + \cdots v_{16}^2) \\ &= \tfrac{1}{8}(1.421). \end{aligned}$$

For the 8 intermediate ordinates

$$v_1^2 + v_3^2 + v_5^2 \cdots v_{15}^2 = 2.692.$$

Hence from equation 76

$$\delta_2 = \frac{2.692 - 1.421}{2.692 + 1.421} \times 100 = 31 \text{ per cent.}$$

Similarly a 16 coefficient analysis gave

$$\delta_2 = \frac{8.347 - 4.113}{8.347 + 4.113} \times 100 = 34 \text{ per cent.}$$

However, a 32 coefficient analysis gave

$$\delta_2 = \frac{12.46 - 11.88}{12.46 + 11.88} \times 100 = 2.4 \text{ per cent.}$$

This represents an error of only 1.2 per cent. in the ordinate reading, which is well within experimental error.

Also, we are interested in the first 16 or 18 coefficients only and these would be influenced by no coefficients under the 48th or 46th. This follows because the first coefficient is altered by the last (63d), the second by the 62d, etc. It seems highly improbable that we should have any harmonics above the 46th, so a further analysis should not be necessary.

To test this conclusion, the analysis for 64 coefficients was performed. It was found that all coefficients above the 38th

TABLE 6.

Interpolation for LiCl at $t = 10.5$.

(B_k are the original and $\bar{B}_k$ the transformed coefficients.)

k.	B_k.		$\bar{B}_k$.	Method 1.		Method 2.	
				P.	PB_k.	$\bar{P}$.	$\overline{PB}_k$.
1	−732	−10590	11322	−.0183	13	−.0183	−207
2	4876	5714	10590	.0376	183	.0193	204
3	−5037	−677	−5714	−.0593	299	−.0217	124
4	−816	1493	677	.0849	−69	.0256	17
5	−4797	3304	−1493	−.1173	563	−.0324	48
6	−2498	−806	−3304	.1616	−404	.0443	−146
7	−1322	2128	806	.2286	302	−.0670	−54
8	−4612	2484	−2128	.3460	−1596	.1174	−250
9	−3227	743	−2484	−.6154	1986	−.2694	669
10	3386	−4129	−743	1.9513	6607	1.3359	−993
11	6404	−2275	4129	2.0465	13106	3.9978	16507
12	3271	−996	2275	−.7110	−2326	1.3355	3038
13	−8	1004	996	.4425	−4	−.2685	−267
14	−1416	412	−1004	−.3265	462	.1160	−116
15	−1848	1436	−412	.2614	−483	−.0651	27
16	−2246	810	−1436	−.2195	493	.0419	−60
17	−1412	602	−810	.1901	−268	−.0294	24
18	−274	−328	−602	−.1683	46	.0218	−13
19	632	−304	328	.1514	96	−.0169	−6
20	1455	−1151	304	−.1379	−201	.0135	4
21	1674	−523	1151	.1269	212	−.0110	−13
22	998	−475	523	−.1176	−117	.0093	5
23	−153	628	475	.1098	−17	−.0078	−4
24	−980	352	−628	−.1029	101	.0069	−4
25	−1016	664	−352				
26	−658	−6	−664	Sum	18984	Sum	18534
27	−279	285	6				
28	168	−453	−285				
29	707	−254	453				
30	886	−632	254				
31	632	0	632				
			0				

$$P = \left(\frac{1}{t-k} - \frac{1}{t+k} \right) (-1)^k$$

$$\bar{P} = \frac{1}{t-k} - \frac{1}{t-(k-1)} - \frac{1}{t+k} + \frac{1}{t+k-1}$$

−11322 (check)
(alternate sum)

$$\frac{10.5 \sin 0.5}{16} = \frac{10.5}{50.27} = 0.2089$$

$$I(10.5) = 18.53 \times 0.2089 = \underline{\underline{3.871}} \text{ at } 3.646 \text{ A}^\circ$$

were completely negligible. By reading the 64 intermediate ordinates and applying the accuracy test the value of δ_2 was found to be 0.5 per cent. for the 64 coefficient analysis.

Interpolation for Intermediate Points.

It is observed that the transform curve $I(t)$ is not completely determined by the equidistant values $I(k)$ shown by the open circles. It would be possible to determine intermediate points by extending the range of the $si(s)$ curve and making a new analysis, but this would be a laborious process. Also we would sacrifice considerable convergence and hence considerable accuracy. A better method is to determine intermediate points by applying the interpolation formula (95) or (101).

The superior convergence of the second method which uses the transformed coefficients $\bar{B}_k$ is shown in Table 6, which gives a comparison of the two methods for $t = 10.5$ (i.e., $r = 10.5(\pi/a)$ where $a = 9.048$). The transformed coefficients give an error of less than one per cent. by using only 7 terms while 24 terms with the original coefficients still give an error of more than two per cent.

For all the interpolated points 10 terms were used. The resulting values are shown on the transform curve by solid circles (Fig. 3).

CONCLUSION.

The analysis for 32 coefficients has been accomplished by making 4 analyses for 8 coefficients. At Purdue we have occasionally found a few more coefficients desirable for good convergence and most of our Fourier analyses have been for 36 coefficients by making 2 analyses for 18 coefficients.

The practical value of the four improvements in Fourier analysis is well demonstrated by their application to the x-ray analysis of molten LiCl. Our density distribution confirms the analysis by Lark-Horovitz and Miller[8] who have given a complete discussion of the distribution curve.

We wish to acknowledge with thanks the helpful criticism of Professor Lark-Horovitz in the preparation of this material.

THE NATURE OF STATIC FRICTION.

BY

WALTER CLAYPOOLE[1] AND DONALD B. COOK.[2]

ABSTRACT.

The factors concerned in friction are discussed, together with a review of the limitations of the various approaches heretofore used. A more satisfactory view of the basic cause of static friction is developed in relation to modern experimental knowledge by considering the pertinent factors in systems having ideal surfaces, then relating this discussion to conditions in the best obtainable practical situation.

INTRODUCTION.

Although extensive experimental work has been done in the field of friction, no completely satisfactory picture of the mechanism of the phenomena has yet been presented. The earliest investigators[3, 4] attributed friction to the interlocking of surface asperities. Modern practice has tended to overlook the possibility that asperities do exist on a sub-microscopic scale. Hardy[5] experimented almost entirely with lubricated surfaces, investigating the effect on the static coefficient of the nature and dimensions of the molecules in the lubricating film. This approach failed to consider the effect of contour irregularities, but the correlation of frictional resistance with the molecular properties of the lubricant is important wherever the protective layer on either surface remains intact. Tomlinson[6] attempted to show that sliding and rolling friction between dry metals could be explained on the basis of the attractive and repulsive forces between atoms in the opposed surfaces.

Bowden and his associates have recently added much to

[1, 2] Fellowship established at Columbia University by The Texas Company.

[3] Amontons, *Hist. Acad. Roy. des Sciences, Mémoires de Mathématique et de Physique*, p. 206 (1699).

[4] Coulomb, *Acad. Roy. des Sciences, Mémoires de Mathématique et de Physique*, p. 161 (1785).

[5] Hardy, Sir W. B., "Collected Works," Camb. Univ. Press, 1936.

[6] Tomlinson, *Phil. Mag.*, 7th ser., 7, 905 (1929).

the experimental knowledge of friction. Bowden and Hughes[7] described a vacuum and high temperature technique which is effective in the removal of the residual oxide and impurities that remain when oily contamination is removed by the best ordinary methods. They found the frictional coefficients between metals so treated to be as much as twenty-fold compared with those for metals tested under ordinary "dry-clean" conditions. Bowden and other co-workers[8, 9, 10] have investigated kinetic friction with special methods showing that "stick-slip" phenomena often occur, associated with the formation of minute welds between opposed surface irregularities. This is valuable confirmation that the latter factor is of primary interest in consideration of friction.

Indeed, the separate approaches have largely overlooked the fact that friction can be completely accounted for not by any one factor but only by a varying combination of a large number of factors, many of which are difficult of control. Among them may be listed the following:

1. Geometrical contour of the surfaces in contact.
2. Departures of surfaces from true geometrical figure.
3. Physical properties of the opposed solids.
4. Molecular forces resident in the solid surfaces.
5. Molecular forces at the free surface of an adsorbed film protecting either solid surface.
6. Effect of polar molecules in the protective film.
7. Length of the hydrocarbon chain in the film.
8. Effect of adsorbed water and gases.
9. Effect of chemically formed films on the solid surfaces.

The choice between determination of the static or kinetic coefficient as an index to friction depends on which is more easily measured in the system studied. It is legitimate to assume that the situation in static friction is more fundamental. In static contact the various forces involved are in equilibrium and depend definitely on the conditions at the contact region before motion is initiated, the larger heat effects of running friction [11] being absent. It is for this reason that, in this inquiry, the emphasis is on the mechanism of static friction. It will be useful first to discuss the probable behavior of simple hypothetical systems for which all conditions are rigorously specified. A discussion of the behavior of less ideal systems will follow, in an attempt to find what changes may be expected when a specified departure occurs from the conditions of the ideal model.

---

[7] Bowden and Hughes, *Proc. Roy. Soc. A*, **172**, 263 (1939).

[8] Bowden and Leben, *Proc. Roy. Soc. A*, **169**, 371 (1939).

[9] Bowden and Tabor, *Proc. Roy. Soc. A*, **169**, 391 (1939).

[10] Bowden and Leben, *Roy. Soc. London, Phil. Trans. A*, No. 799, **239**, 1–27 (1940).

I. AN IDEAL MODEL. PLANE SURFACES.

As a hypothetical system for the study of friction, consider for example two similar disks made of hard and homogeneous material, and having geometrically plane contacting surfaces. Let the system be under absolute vacuum, so that the effect of residual contaminations can be neglected.

(1) *Contact Surfaces Unprotected.* If the two plane surfaces are brought together along a line normal to each, true contact will occur over the whole area. The atomic fields of the surface molecules will interpenetrate, thus creating a joint of strength comparable with that of the bulk solid.

Experiment.—In an easily reproducible experiment, two wires, one of platinum, the other of platinum-rhodium, were cold-welded after thorough cleaning with $KHSO_4$ at the highest temperature afforded by an air-gas blast lamp. To ensure virtually complete removal of all contamination it was necessary, towards the end of the process, so to hold the wires that the molten flux ran down from the ends which were to be joined. When cold, the cleaned ends were laid across each other and lightly tapped with a hammer, the object being to flow the metal and bring a relatively large area into intimate contact.

The attached photographs, taken at a magnification of 1000× show a section across the joint after polishing and etching. The thin straight lines are polish scratches, and the dark irregular areas are foreign inclusions, probably oxide.

One end of the weld is shown in Fig. 1, the dark line indicating very close contact but something less than true molecular union. A little further away from the end the junction has become more intimate, as shown in Fig. 2, while still further along, complete fusion is shown in Fig. 3. The photograph in Fig. 4 is the other end of the weld, and here, the broken line indicates localities where, also, complete fusion has occurred.

---

[11] Bowden and Ridler, *Proc. Roy. Soc. A*, **140**, 640 (1936).

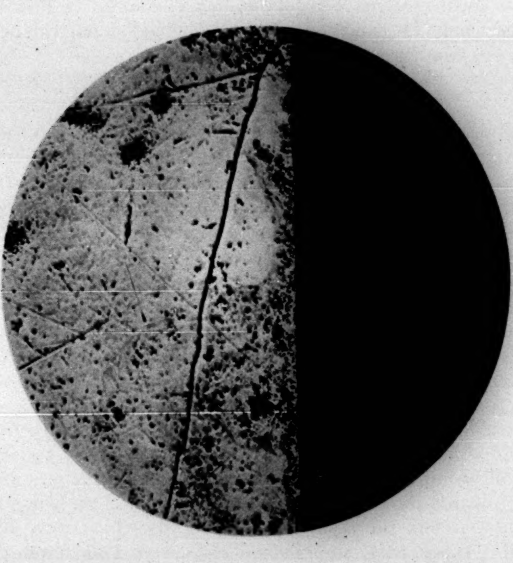


FIG. 1. One end of cold-weld joint showing close contact between platinum and platinum wires. Magnification 1000×.

(2) *Contact Surfaces Each Overlaid with an Oriented Monolayer.* Allow the surfaces to come together as before, but with the additional specification that the temperature be absolute zero. Under these conditions the free ends of the adsorbed molecules will lie in a true plane, and in the final position of equilibrium when the surfaces are brought together, the separation of the opposed hydrocarbon ends will be somewhat greater than 2 Å, which is the order of the

C-C spacing in such molecules. The opposed surfaces can thus be regarded in some degree as "floating" on each other in a molecular field of force. If one surface is moved tangentially with respect to the other, the field of force is disturbed, vibrational energy being imparted to the molecules of the opposed films. Since these molecules are strongly bonded to the solid surfaces, energy is absorbed by the latter. The work of friction represented by this dissipated energy is at most, however, very small, since it is ultimately referable to the relatively small forces of chemical affinity between the hydrocarbon groups forming the free surfaces of the opposed films. Static friction will thus be much less than in any practical approximation of this model.

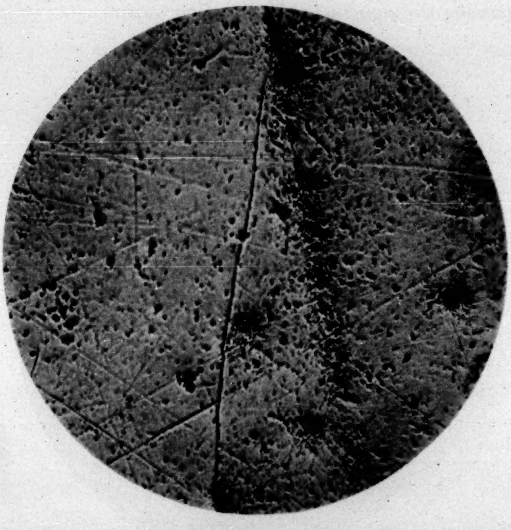


FIG. 2. Section further along the joint showing more intimate contact than in Fig. 1. Magnification 1000×.

II. AN IDEAL MODEL. SPHERICAL SURFACES.

Suppose the two plane faces of the opposed surfaces in the former case have a number of elevated areas of truly spherical contour and small height. Let their tops lie in a true plane, and let their number and distribution on each surface be identical. This provision permits the two elements to be so brought together that opposed spherical contours meet on a line passing through their centers of curvature.

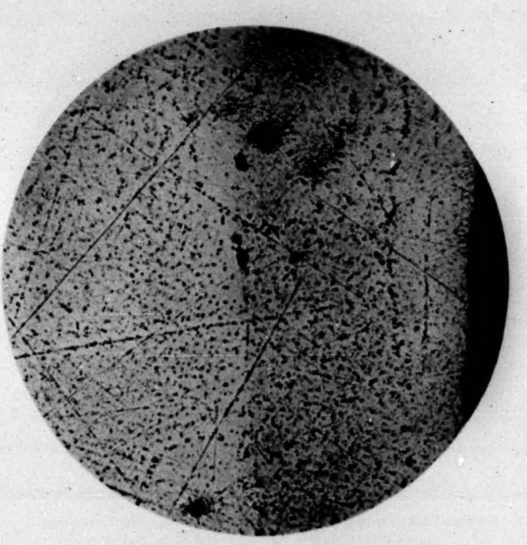


FIG. 3. Complete fusion indicated in this area. Magnification 1000×.

(1) *Contact Surfaces Unprotected.* If the elements are brought together under the conditions specified above, and if, in addition, the line of approach is normal to both planes, all contacts will be made simultaneously, and the load will be equally divided among them. If the number of contacts is large enough, elastic deformation only will occur, but if the number is small, the flow pressure of the material may be

reached. In either case, under the specified conditions, the contact areas will be circular and will lie in a common plane.

There is now no essential difference between the flat-contact model and the spherical-contact model. In the complete absence of surface contamination the atomic fields of the contacting solids interpenetrate, and a strong union may be expected.

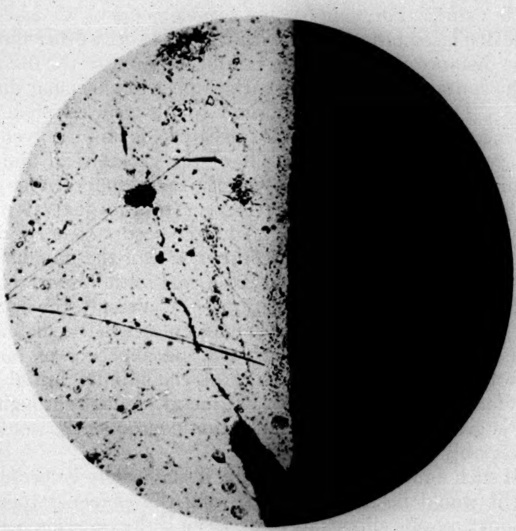


FIG. 4. Other end of weld. Localities are indicated along the broken line where complete fusion has occurred. Magnification 1000×.

(2) *Contact Surfaces Each Overlaid with Oriented Monolayer.* If the approach and contact are carried out as before, consider what occurs at the contact region between any given pair of spherical surfaces. If elastic deformation only occurs, the Hertz formula will give the area of the contact circle. This area is a true plane normal to the load line. If the conditions are such that the flow pressure of the material is

reached, the only permitted change in the contact area will, under the specified conditions, be dimensional. If the approach and contact be carried out without any side slip whatever, it is improbable that any appreciable molecular contact between the solid surfaces will occur. Thus, the initial static friction will be small, as in case I, section 2.

As soon, however, as an initial slip so small as to be hardly observable occurs, the friction immediately rises to an appreciable value. For, suppose the two elements of the assembly to be displaced tangentially with respect to each other through some distance which, though small compared to the diameter of the contact circle, is large with respect to molecular dimensions. The original line, normal to the general plane and passing through the centers of curvature, is now resolved into two parallel lines. As a result of the geometrical and mechanical unbalance of the system, the individual contact areas are now slightly tilted with respect to the general plane. Plastic flow of the material no longer occurs strictly in the general plane, and a further consequence of the unbalance is that the deformation is unsymmetrical with respect to the contours of the spherical elevations. As a result, the orientation of the protecting layer of molecules will be disturbed, with a resultant tendency toward the exposure of minute unprotected areas. As a result of the minute initial slip, therefore, the possibility of clean metal contact through the film is greatly increased, and friction consequently would be appreciably higher.

If such a model could be made, it may safely be presumed that it would be impossible to bring the elements together without any side slip whatever. Even if the experiment were made under the best possible conditions, unavoidable mechanical vibration would prevent absolutely static first contact from remaining permanent.[12]

III. A PRACTICAL MODEL.

Consider now a friction system made up of two surfaces which are as flat and smooth as any obtainable in workshop practice. Such surfaces are, of course, very far from being geometrically plane. Asperities may be greatly reduced by polishing, but plateaus and valleys are certainly present. The average height of these plateaus above the median plane is probably not less than 1 micro-inch, while their width is much greater, perhaps of the order of several thousandfold.

---

[12] On the significance of small residual vibrations and their effect on surfaces, see Tomlinson, *Inst. Mech. Eng. Jnl.*, 141, no. 3 (1939).

When the units of such an assembly are brought together, the area of contact will clearly be very much less than in the ideal case of geometrical planes—where the actual and apparent areas of contact are identical. In the practical model, the plateaus and valleys are distributed at random, and the elevations vary in height.

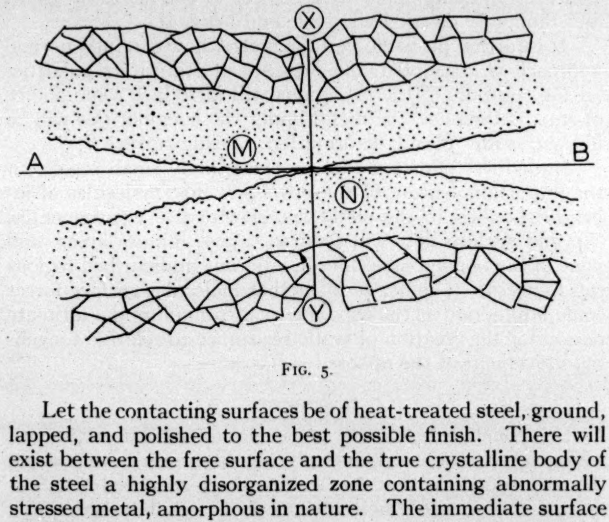


FIG. 5.

Let the contacting surfaces be of heat-treated steel, ground, lapped, and polished to the best possible finish. There will exist between the free surface and the true crystalline body of the steel a highly disorganized zone containing abnormally stressed metal, amorphous in nature. The immediate surface (Beilby layer) will be distinctly amorphous and will carry oxides and general contamination. Let the contacting surfaces be overlaid each by a monolayer of oriented molecules. Consideration of an unprotected surface case is omitted, since true freedom from contamination is impossible to achieve in practice.

In Fig. 5 the vertical scale of which is considerably exaggerated, *M* and *N* represent the tops of two opposed elevations, not exactly opposite each other, which have been brought together along the line of approach XY, normal to the reference plane which is seen in cross section as the line AB.

It may safely be assumed that the elevations on a polished surface are relatively spherical in contour, but it is not legitimate to assume that these contours are geometrically smooth. The presence of tiny irregularities on the surface of the elevations has been indicated in the diagram. Another departure from the ideal model with spherical contacts lies in the low probability that elevations meet "head on." In this present model, operating under practical conditions, the planes of contact are randomly tilted with respect to the reference plane, and randomly distributed above and below it.

It is further probable that, no matter how carefully normal approach is attempted, the presence of minute irregularities on the contours of the elevations will introduce some degree of slip. Moreover, the metal under the irregularities will be likely to suffer plastic deformation, causing surface flow.

Both these effects are on an exceedingly small scale, but the important feature of the process is that molecules of lubricant bonded to the metal travel with it. Consequently, slip and surface flow will tend to expose minute areas, and none of the bonded molecules in adjacent undisturbed regions can move in. The net result is that molecular surface forces come into action at the exposed areas, providing the ultimate reason for the creation of static resistance to relative tangential movement of the opposed surfaces.

IV. SUMMARY.

It is concluded that true plane surfaces each overlaid with a monolayer of oriented molecules would form a system where the only friction is that due to the energy absorbed in vibration of the oriented molecules as they are disturbed. The behavior of a system with true spherical contacts would be similar, except that the ideal minimum would be raised by the least relative side-slip of the surfaces, resulting from non-normal approach or from residual vibrations.

The best possible practical flats are very far from being plane, carrying small elevations large with respect to molecular dimensions. It is not legitimate to assume that these elevations are themselves geometrically smooth, and it is postulated that the minute surface irregularities present on them are responsible for some degree of side-slip, even when the maximum care is taken to bring the opposed surfaces together normally. Where side-slip does occur, and where the pressure is high enough to cause plastic flow, the tendency will be to expose minute areas at which molecular surface forces come into play, creating the resistance to applied tangential force which accounts for static friction.

In conclusion, the authors wish to express their appreciation for helpful suggestions to Prof. G. B. Karelitz, Prof. T. B. Drew, and Mr. R. P. Graham of Columbia University, and to Dr. L. C. Roess and other members of the research staff of The Texas Co.

New Insulating Material.—(*Compressed Air Magazine*, Vol. 47, No. 3.) Something new in insulating materials was discovered accidentally while experimenting with perlite, which is described as volcanic glass with a concentric, shelly texture and, usually, a notable percentage of water. In investigating the material with the thought of possibly using it in making enamels and other infusible products, L. LEE BOYER noticed that when perlite was exposed to heat well below its melting point it exploded like popcorn and formed a mass lighter than cork. Upon examination under a microscope it was found that the product had a porous structure and that the pores were well sealed. Subsequent research revealed that it possessed insulating properties of value. Near a large deposit of perlite in Superior, Ariz., has been erected a semicommercial plant consisting of a gas-heated cylindrical kiln lined with firebrick. Before the material is fed into one end of it, it is first crushed, and in that form explodes in from 30 to 45 seconds when subjected to a temperature of 1650 deg. F. In less than a minute from the time it enters the kiln, the mass is discharged into a Dorr classifier where the particles of perlite, now 6½ to 7 times their original volume, float while the pieces that did not expand sink to the bottom. When dried and ready for the market the product is pure white, fluffy, and free of impurities. According to conductivity tests, perlite has insulating properties equal to those of ground cork, rock wool, asbestos, vermiculite, and similar substances. It is claimed to be an especially good fire-resistant material for construction purposes because it will not absorb water and is infusible up to a temperature of 3000 deg. F. When ground to 40–80 mesh it is suitable for filtering water; and, with the addition of a small percentage of sodium silicate, forms a light fire-resistant coating that adheres to wood, paper, and metal. It is estimated that the insulating material can be made at a reasonable cost, and its production on a commercial scale is soon to be started.

R. H. O.

HISTORICAL NOTE ON THE DEFLECTING INFLUENCE OF THE ROTATION OF THE EARTH.

BY

EDGAR WILLIAM WOOLARD, Ph.D.,

U. S. Weather Bureau, Washington, D. C.

The results of the operation of familiar physical laws in the atmosphere are frequently somewhat at variance with what is to be expected on the basis of ordinary experience, both because of the very great differences between conditions in the laboratory and in the free atmosphere, and also because of the vast scale on which atmospheric processes take place; many factors which ordinarily are of slight consequence in physical occurrences and to which, therefore, not much emphasis is usually given, become of dominant importance in meteorological phenomena.[1] One of the most striking and fundamental examples is the effect of the rotation of the earth in determining the principal characteristics of the major atmospheric circulations.

This effect depends upon the principle that, because of the rotation of the earth with angular speed ω, a particle of mass m moving horizontally *in any direction* with speed v relative to the surface of the earth at latitude φ has an acceleration (relative to this surface) perpendicular to the direction of motion and toward the right in the northern hemisphere, toward the left in the southern, of magnitude $2mv\omega \sin \varphi$, independent of and in addition to all accelerations that it may happen to have because of forces acting on it.[2] This principle was first formulated by Ferrel in 1856, although it could have been deduced long before from general equations of motion that had been derived by Laplace and others.

As a consequence of this kinematic deflection, the wind direction in a steady frictionless horizontal circulation of air on a large scale (such as is closely approximated in the upper atmosphere) is perpendicular to the barometric pressure gradient, i.e., *along isobars*.

---

[1] Several interesting examples are discussed by W. J. Humphreys, "Weather Proverbs and Paradoxes," 2 ed., Baltimore, 1934.

[2] See, e.g., W. J. Humphreys, "Physics of the Air," 3 ed., pp. 114–116. New York, 1940.

The establishment of the fundamental laws of atmospheric circulation is thus a chapter in the historical development of the general dynamical theory of relative motion. The abstract kinematics and dynamics of relative motions [3] attracted attention early in the history of analytical mechanics—e.g., Clairaut carried out an important investigation in 1742; meanwhile, it had long been recognized that the peculiar features of several natural phenomena were to be ascribed to the fact that these phenomena involve motions relative to the rotating earth, and attempts were soon made to construct their physical explanations on the basis of the mathematical theory of relative motion.

The particular effect of the rotation of the earth that first received attention was the deviation of freely falling bodies from the vertical; although discussed by Newton and Hooke in 1679, and the subject of a large number of investigations from the early 18th century on, this problem did not receive a final solution until the work of Roever and Moulton in the early 20th century.[4] That the rotation of the earth was the cause of some of the characteristics of the prevailing winds was first realized by Hadley (1685–1768), in 1735, while in 1740, Colin Maclaurin (1698–1746) recognized that it was a factor in oceanic tidal motions, but in neither case were the ideas dynamically complete.

During the 19th century, the investigation of the dynamical theory of motion relative to the earth was stimulated by several further phenomena which at the time were of considerable importance and interest, notably the Foucault pendulum experiment (1851) and the Perrot experiment (1859) for demonstrating the rotation of the earth, and "Baer's Law" relating to the effect of the earth's rotation on the erosion of river banks (apparently first formulated by P. A.

---

[3] See E. J. Routh, "Dynamics of a System of Rigid Bodies," Part II, chap. 1, 6 ed., London, 1905.

[4] See F. R. Moulton, "Differential Equations," pp. 155–156 (New York, 1930).

Slavsov in 1844, and later by others—see *Science*, **76**: 516, 1932). Meanwhile, general theories of the motion of both a particle and a fluid relative to the earth had been formulated, and were available for application to any specific case: Laplace had, by 1775, constructed a dynamical theory of the tides, in which the effect of the rotation of the earth was an essential element.[5] Also, both Laplace ("Méc. Cél.," Bk. X, ch. 5) and Gauss (in 1803—see *Werke*, Bd. V, Göttingen 1877, pp. 495–503) developed the general equations for the motion of a particle relative to the rotating earth, and applied them to the motion of a freely falling body; while Poisson, in 1837, gave a complete discussion of the general motion of a particle relative to the rotating earth, in connection with a theory of the motions of projectiles.[6] Furthermore, Coriolis, in a series of papers from 1832 to 1836, had constructed an abstract theory of relative motion in general;[7] and in modern meteorological literature, the deflection is usually referred to as the "Coriolis effect."

Applications of these general mathematical theories to atmospheric circulations were slow in being made, however; and it is significant that the initial investigations in dynamical meteorology were not made by meteorologists: Probably it was not an accident that the first contributions to the dynamical theory of atmospheric motions were made by an investigator whose principal work was the mathematical theory of the tides, viz. Wm. Ferrel (1817–1891), who was the first to derive a complete quantitative formulation of the effect of the rotation of the earth on winds, from the mathematical theory of fluid motion on the rotating earth. Ferrel was, of course, thoroughly familiar with Laplace's work, but to what extent he was at first acquainted with the other writings cited above is not clear.

---

[5] "Méc. Cél.," Bks. IV, XIII, and Bk. I, ch. 8. The general equations of motion for the ocean and for the atmosphere are developed, and applied to both oceanic and atmospheric tides.

[6] S. D. Poisson, "Extrait de la première partie d'un Mémoire sur le mouvement des projectiles dans l'air, en ayant égard à leur rotation et à l'influence du mouvement diurne de la Terre," *Comptes Rendus*, **5**: 660–667, 1837; S. D. Poisson, "Mémoire sur le mouvement des projectiles dans l'air en ayant égard à la rotation de la terre," *Journal de L'École Royale Polytechnique*, Tome 16, Cahier 26, pp. 1–68, 1838 (see Abbe, "*Mechanics of the Earth's Atmosphere*," Third Collection, pp. 8–15). The existence of an effect of the earth's rotation on artillery fire was explicitly realized during the Civil War—see M. W. Humphreys, *Jour. U. S. Artillery*, **2** (1893), p. 583—although with the imperfect equipment then in use no correction could be made for it.

[7] G. Coriolis, "Sur le principe des forces vives dans les mouvements relatifs des Machines." *Journal de L'École Polytechnique*, Cahier 21, Tome 13, pp. 268–302, 1832. G. Coriolis, "Sur le manière d'établir les différens principes de mécanique pour des systèmes de corps en les considérant comme des assemblages de molécules." *Journ. de L'École Polytechnique*, Cahier 24, Tome 15, pp. 93–125, 1835. G. Coriolis, "Mémoire sur les équations du mouvement relatif des systèmes de corps." *Journ. de L'École Polytechnique*, Cahier 24, Tome 15, pp. 142–154, 1835. See also *C.R.*, **2**: 172–174, 1836.

Several of Ferrel's predecessors had obtained more or less incomplete or inexact results by general reasoning, although comparatively little had been added to the ideas about atmospheric circulation since Hadley's attempted explanation of the trade winds. This explanation,[8] which occurred independently to others also, was slow in becoming widely known, and was dynamically deficient. Meanwhile, empirical knowledge was accumulating; and during the early 19th century synoptic maps began to be drawn. It was established by Benjamin Franklin and later investigators that current weather conditions travel over the earth in a fairly regular manner, altering more or less as they move; and theories of the nature of storms were formulated, the more notable being those of Dove, Redfield, and Espy. Dove, in taking the rotation of the earth into account in his theory of storms,[9] proceeded on the erroneous principle that if, from any cause, air "receives an impulse to flow in a parallel circle, the rotation of the earth will not have any influence upon it . . . ." Neither Redfield nor Espy recognized the rotation of the earth as a factor in the wind circulations in storms; and the polemic controversy that developed between them might have been avoided had more attention been given to a remarkable little paper by Tracy which appeared in 1843 and which gave the earliest essentially correct qualitative discussion of the effect of the rotation of the earth on the directions of winds.[10]

---

[8] George Hadley, "On the Cause of the General Trade Winds," *Phil. Trans.*, **39**: 58, 1735; *Abr. Phil. Trans.*, **8**: 19–22. Reproduced in facsimile by Hellmann, *Neudrücke von Schriften und Karten*, No. 6, Berlin 1896; reprinted in Cleveland Abbe, "Mechanics of the Earth's Atmosphere," Third Collection (*Smithsonian Misc. Coll.*, **51**, No. 4).

[9] See H. W. Dove, "On the Influence of the Rotation of the Earth on the Currents of its Atmosphere," *Phil. Mag.* (3), **11**: 227–239, 353–363, 1837 (from *Pogg. Ann.*, 1835).

Ferrel's earliest published paper was a non-mathematical essay that appeared in 1856;[11] it was followed a few years afterwards by a complete mathematical treatment,[12] in which Ferrel gave the earliest mathematical explanation of the famous Buys Ballot's Law (empirically formulated independently and almost simultaneously by Buys Ballot and Erman), and first accounted for the high pressure belts, the equatorial pressure depression, and the rotary motion of storms. Later, Ferrel published a still more comprehensive and elaborate theory of atmospheric circulations, general and secondary.[13] All of Ferrel's earlier papers were subsequently reprinted by the U. S. Signal Corps,[14] and Ferrel himself gave later expositions in his two textbooks, "Recent Advances in Meteorology" (Ann. Rept. Chief Signal Officer, 1885, Appendix 71), and "Popular Treatise on the Winds" (New York, 1889); the latter, as a non-mathematical exposition of an intricate subject, deserves to rank with G. H. Darwin's notable book on *The Tides*.

Accumulated data of observation and further theoretical investigation since Ferrel's time have shown that in large part his theories of the planetary circulation, and of the structure and mechanism of extratropical cyclones, are untenable—in particular, his scheme of planetary circulation is far too simple, and is inconsistent with the actual circulation at high levels; but the fundamental dynamical *principles* which he formulated are permanent contributions to meteorology, and his pioneer investigations were of fundamental importance for the early development of dynamical meteorology.

---

[10] Charles Tracy, "On the Rotary Action of Storms," *Amer. Jour. Sci. and Arts*, **45**: 65–72, 1843. Reprinted in Abbe, "Mechanics of the Earth's Atmosphere," Third Collection.

[11] Wm. Ferrel, "An Essay on the Winds and the Currents of the Ocean." *Nashville Journal of Medicine and Surgery* (ed. W. K. Bowling), Oct. and Nov. 1856.

[12] Wm. Ferrel, "The Motions of Fluids and Solids on the Earth's Surface," *Runkle's Mathematical Monthly*, 1858–1860; a small edition was reprinted in quarto pamphlet form in 1860. See also: Wm. Ferrel, "The Motions of Fluids and Solids Relative to the Earth's Surface," *Amer. Jour. Sci.* (2), **31**: 27–51, 1861; and Wm. Ferrel, "Relation between the Barometric Gradient and the Velocity of the Wind," *Amer. Jour. Sci.* (3), **8**: 343–362, 1874.

[13] Wm. Ferrel, "Meteorological Researches for the Use of the Coast Pilot," in *Repts. U. S. Coast and Geodetic Survey* for 1875, 1878, 1881.

[14] *Professional Papers of the Signal Service:* No. VIII, Recent Mathematical Papers concerning the Motions of the Atmosphere, Part I, Washington 1882; No. XII, Popular Essays on the Movements of the Atmosphere by Wm. Ferrel, Washington 1882.

ELECTRODE SHAPING TO ESTABLISH A UNIFORM ELECTRIC FIELD ALONG HIGH VOLTAGE RESISTORS.

BY

PAUL SHALLCROSS,

Moore School of Electrical Engineering of the University of Pennsylvania.

ABSTRACT.

An electrode design is developed for the purpose of obtaining a constant potential gradient along the surface of a cylinder. Such a design is important in a high voltage low current resistance in order that the voltage drop per unit of resistance be constant. It is found that the dimensions of such a resistor and its associated electrodes can be reduced considerably by the proper electrode design.

In the design of high voltage resistors it is often desirable to dissipate as small an amount of power as may be economically practical. This may be required when the total amount of power available is small to prevent an excessive drop of the supply voltage. However, when the resistance is very high the effect of capacitance may be considerable. In the case of d.c. voltages this would produce electrical stress in the air and insulation in the vicinity of the high voltage resistor and may produce damage to the resistor itself from corona. In the case of a.c. voltages there may be an additional effect of displacement currents which may be quite appreciable in comparison to the normal high voltage resistor current. These effects will act to distribute the voltage drops unequally along the resistor. This causes errors in the use of resistors as high voltage multipliers in case of both a.c. and d.c. measurements.

It has been noted in the literature that when high voltages are encountered, the distribution of potential must be controlled to avoid undesirable behavior. Bennett and Frendenhall [1] suggest that high voltage insulators have their surface coated with a resistance material to make the potential gradient more uniform, thereby increasing insulator life or rating.

---

[1] E. Bennett and G. Frendenhall, *Electrical Engineering*, Vol. 54, pp. 1084–1087.

In a study of a high voltage a.c. multiplier resistor composed of a number of shielded series resistance units, Davis,[2] has called attention to the need of the correct adjustment of the shield potential as an important consideration in the design of a high voltage resistor having a small phase angle at both the low and high voltage ends.

When the high voltage multiplier resistor is to be used with a restricted source of power it is desirable to use as small a current as may reasonably well be measured. However, with small current flow there is more likelihood of trouble from current because of distributed capacity. In addition, high potential gradients result in corona currents. These additional currents flowing in only part of a high voltage multiplier result in an incorrect indication of voltage and may also cause failure of the multiplier itself.

The author has noted failure due to corona, of several experimental resistors about one-quarter to one-third the distance to ground from the high voltage end.

The object of this paper is to describe the design of an electrode system which will enable a high resistance such as a voltmeter multiplier to be used for high voltages without the use of a series of potential shields, each at a different potential and with a minimum of deleterious effect of corona on the surface of the resistor. This is to be accomplished by designing the electrode system so that along the surface of a cylinder, having a common axis with the electrode system, a uniform fall of potential results. A voltmeter multiplier having these characteristics should have considerable application in the measurement of X-ray tube potentials, since here the current taken by a voltmeter multiplier and the potential divider supplying the potential shields of its guard circuit can easily be as large as that required by the tube itself. The use of an electrode system in place of a guard circuit would result in several economies. The multiplier would be less expensive and the power supply would deliver less current and thus be cheaper. Inasmuch as less current would be required the cost of operation would also be reduced.

---

[2] R. Davis, *Journal of Scientific Instruments*, Vol. 5, pp. 305–312 and pp. 354–361.

In the mathematical treatment of this problem it is assumed that the cylinder supporting the resistance is thin, so that but little distortion of the field will occur because of the stressing of the dielectric of the cylinder.

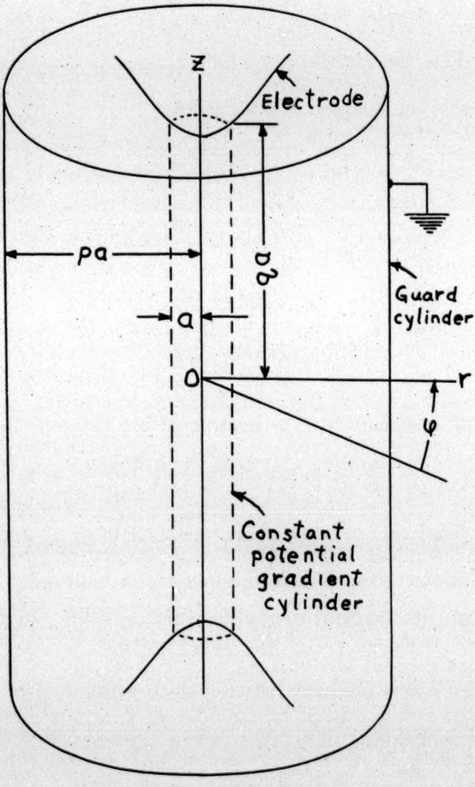

Z
Electrode
pa
pb
a
O
r
φ
Guard cylinder
Constant potential gradient cylinder

FIG. 1. General diagram.

In order to prevent changes in the external field resulting in disturbances on the surface of the cylinder, recourse will be made to the use of an external grounded cylinder hereinafter called the guard cylinder, as shown in Fig. 1.

Laplace's equation in cylindrical coördinates states that:[3]

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2} = 0, \tag{1}$$

where V is the electric potential, r, z, and φ are shown in Fig. 1.

Because of symmetry about Z axis:

$$\frac{\partial^2 V}{\partial \varphi^2} = 0,$$

so

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0. \tag{2}$$

Assume

$$V_{(r,z)} = R_{(r)}Z_{(z)}.$$

Then

$$Z\frac{\partial^2 R}{\partial r^2} + \frac{Z}{r}\frac{\partial R}{\partial r} + R\frac{\partial^2 Z}{\partial z^2} = 0. \tag{3}$$

Dividing by RZ

$$\frac{1}{R}\frac{\partial^2 R}{\partial r^2} + \frac{1}{Rr}\frac{\partial R}{\partial r} = -\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}. \tag{4}$$

As left hand side of equation is independent of Z, and right hand independent of R, each at most is equal to a constant. The solutions are subject to the following conditions:

(1) V and its gradient are finite, varying from $+E$ at one electrode to $-E$ at the other electrode.
(2) When $r = pa$, $V = 0$.
(3) When $z = 0$ (midway between electrodes), $V = 0$,
(4) When $r = a$ (radius of cylinder of constant gradient), $V = \frac{zE}{qa}$.

---

[3] W. E. Byerly, "Fourier's Series and Spherical Harmonics," p. 226.

If the constant of separation is ω^2, equation (4) can be written as two equations:

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} - \omega^2 R = 0 \tag{5}$$

and

$$\frac{d^2Z}{dz^2} + \omega^2 Z = 0. \tag{6}$$

The general solution for R is:[4]

$$R = CI_0(\omega r) + cK_0(\omega r),$$

$I_0(\omega r)$ and $K_0(\omega r)$ being Bessel functions with imaginary argument of the first kind and second kind and of zero order respectively.

From condition (1) $c = 0$ as $K_0(\omega r) = \infty$ when $\omega r = 0$.
From condition (2) $C = 0$ as $I_0(\omega r)$ is always a real positive number.

Therefore R is identically equal to zero and this general solution is valueless for these conditions. Thus the constant may not be positive or zero.

If the constant of separation is $-k^2$, k being a real constant, equation (4) can be written as two equations:

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + k^2 R = 0 \tag{7}$$

and

$$\frac{d^2Z}{dz^2} - k^2 Z = 0. \tag{8}$$

The general solutions are:

$$\begin{aligned} R &= DJ_0(kr) + dN_0(kr), \\ Z &= F\sinh(kz + \theta), \end{aligned} \tag{9}$$

$J_0(kr)$ and $N_0(kr)$ being Bessel functions with real argument of the first kind and second kind and of zero order respectively.

From condition (1), $d = 0$ since $N_0(kr) = \infty$ when $kr = 0$

---

[4] The Encyclopaedia Britannica, 11th Edition. Article on "Spherical Harmonics."

and Z is not identically zero. Were d not equal to zero, V would be infinite along the axis where z is not zero.

Therefore $R = DJ_0(kr)$.

From condition (2) there are n possible values of R

$$R_n = D_n J_0(k_n r), \tag{10}$$

K_n being any root of $J_0(k_n pa)$.

The first ten roots of $J_0(k_n pa)$ being[5]

n	root
1.	2.4048
2.	5.5201
3.	8.6537
4.	11.7915
5.	14.9309
6.	18.0711
7.	21.2116
8.	24.3525
9.	27.4935
10.	30.6346

From condition (3) θ of Eq. (9) is zero or $Z = F \sinh kz$ and being restricted to nth terms

$$Z_n = F_n \sinh k_n z. \tag{11}$$

Combining equation (10) and equation (11),

$$V_n = A_n J_0(k_n r) \sinh k_n z \tag{12}$$

or

$$V = \sum_{n=1}^{\infty} A_n J_0(k_n r) \sinh k_n z. \tag{13}$$

It remains to satisfy condition (4). When $r = a$, $V = \dfrac{zE}{qa}$,

$$\frac{zE}{qa} = \sum_{n=1}^{\infty} A_n J_0(k_n a) \sinh k_n z. \tag{14}$$

To simplify, $k_n a$ will be written m_n and $A_n J_0(k_n a)$ will be written B_n. Sinh $m_n(z/a)$ may be expanded, the first few terms being

---

[5] Jahnke-Emde, "Funktionentafeln," p. 327. See also pp. 228–278.

$$m_n \frac{z}{a} + \frac{\left(m_n \frac{z}{a}\right)^3}{3!} + \frac{\left(m_n \frac{z}{a}\right)^5}{5!} \cdot$$

Using this expansion and dividing each side of equation (14) by z/a then to a close degree of approximation

$$\frac{E}{q} = B_1 \left[m_1 + \frac{m_1{}^3}{3!}\frac{z^2}{a^2} + \frac{m_1{}^5}{5!}\frac{z^4}{a^4} + \frac{m_1{}^7}{7!}\frac{z^6}{a^6} + \frac{m_1{}^9}{9!}\frac{z^8}{a^8} + \frac{m_1{}^{11}}{11!}\frac{z^{10}}{a^{10}} \right.$$
$$\left. + \frac{m_1{}^{13}}{13!}\frac{z^{12}}{a^{12}} + \frac{m_1{}^{15}}{15!}\frac{z^{14}}{a^{14}} + \frac{m_1{}^{17}}{17!}\frac{z^{16}}{a^{16}} + \frac{m_1{}^{19}}{19!}\frac{z^{18}}{a^{18}} \right] \quad (15)$$

plus similar terms for the other values of n, using $n = 1$ to 10.

By collecting terms of like order of z/a

$$\begin{aligned}
\frac{E}{q} &= B_1m_1 + B_2m_2 + B_3m_3 + B_4m_4 + B_5m_5 + B_6m_6 \\
&\qquad + B_7m_7 + B_8m_8 + B_9m_9 + B_{10}m_{10}, \\
0 &= B_1m_1{}^3 + B_2m_2{}^3 + B_3m_3{}^3 + B_4m_4{}^3 + B_5m_5{}^3 + B_6m_6{}^3 \\
&\qquad + B_7m_7{}^3 + B_8m_8{}^3 + B_9m_9{}^3 + B_{10}m_{10}{}^3, \\
0 &= B_1m_1{}^5 + \cdots, \qquad\qquad (16) \\
0 &= B_1m_1{}^7 + \cdots, \\
0 &= B_1m_1{}^9 + \cdots, \\
0 &= B_1m_1{}^{11} + \cdots, \\
0 &= B_1m_1{}^{13} + \cdots, \\
0 &= B_1m_1{}^{15} + \cdots, \\
0 &= B_1m_1{}^{17} + \cdots, \\
0 &= B_1m_1{}^{19} + \cdots.
\end{aligned}$$

Having ten equations with ten unknowns it is possible to evaluate the coefficients and

$$\begin{aligned}
\frac{E}{q} &= 3.075{,}9m_1 - 0.649{,}67m_2 + 0.200{,}67m_3 \\
&\quad - 0.064{,}569m_4 + 0.018{,}817m_5 - 0.004{,}586{,}9m_6 \qquad (17) \\
&\quad + 0.000{,}878{,}2m_7 - 0.000{,}122{,}52m_8 \\
&\quad + 0.000{,}011{,}018m_9 - 0.000{,}000{,}477{,}7m_{10}.
\end{aligned}$$

In general, remembering $m_n = k_n a$, throughout the space $r = 0$ to $r = pa$ and from one electrode to the other electrode ($V = +E$ to $V = -E$),

$$
\begin{aligned}
V = 3.075{,}9 \, & \frac{J_0(k_1 r)}{J_0(k_1 a)} \sinh k_1 z \\
& - 0.649{,}67 \frac{J_0(k_2 r)}{J_0(k_2 a)} \sinh k_2 z \\
& + 0.200{,}67 \frac{J_0(k_3 r)}{J_0(k_3 a)} \sinh k_3 z \\
& - 0.064{,}569 \frac{J_0(k_4 r)}{J_0(k_4 a)} \sinh k_4 z \\
& + 0.018{,}817 \frac{J_0(k_5 r)}{J_0(k_5 a)} \sinh k_5 z \\
& - 0.004{,}586{,}9 \frac{J_0(k_6 r)}{J_0(k_6 a)} \sinh k_6 z \\
& + 0.000{,}878{,}2 \frac{J_0(k_7 r)}{J_0(k_7 a)} \sinh k_7 z \\
& - 0.000{,}122{,}52 \frac{J_0(k_8 r)}{J_0(k_8 a)} \sinh k_8 z \\
& + 0.000{,}011{,}018 \frac{J_0(k_9 r)}{J_0(k_9 a)} \sinh k_9 z \\
& - 0.000{,}000{,}477{,}7 \frac{J_0(k_{10} r)}{J_0(k_{10} a)} \sinh k_{10} z.
\end{aligned}
\tag{18}
$$

It now remains to choose definite values for p and q. There is little restriction regarding the value of p. It must be greater than one, and, if large, the overall size of guard cylinder might become excessive.

However the solution, after selection of p, becomes discontinuous for moderate values of q.

While all possibilities were not explored it was found that with a value of $p = 9$, a value of $q = 5.4$ could be realized. With $p = 9$,

$$
\begin{aligned}
V = 3.131 & J_0(.2672r) \sinh .2672z \\
& - 0.715{,}4 J_0(.6133r) \sinh .6133z \\
& + 0.256{,}6 J_0(.9615r) \sinh .9615z \\
& - 0.105{,}0 J_0(1.310r) \sinh 1.310z \\
& + 0.044{,}63 J_0(1.659r) \sinh 1.659z \\
& - 0.020{,}92 J_0(2.008r) \sinh 2.008z \\
& + 0.034{,}84 J_0(2.357r) \sinh 2.357z \\
& + 0.000{,}845{,}0 J_0(2.706r) \sinh 2.706z \\
& - 0.000{,}039{,}61 J_0(3.055r) \sinh 3.055z \\
& + 0.000{,}001{,}309 J_0(3.406r) \sinh 3.406z.
\end{aligned}
\tag{19}
$$

By calculation of V for representative values of r and z it is possible to select various combinations of r and z which all result in an identical value of V. These points may all lie on the surface of an electrode. Similarly the use of $-z$ with the chosen combinations of r will result in an identical value of $-V$.

As these equations meet all the required conditions, then the use of these electrodes will also satisfy the required conditions, that V is finite, varying from $+E$ at one electrode to $-E$ at the other electrode, that $V = 0$, when $r = pa$ that $V = 0$ when $z = 0$ (midway between electrodes) and that $V = zE/qa$ when $V = a$ (radius of cylinder of constant gradient). Reference should be made to Fig. 2 for case $p = 9$, $q = 5.4$.

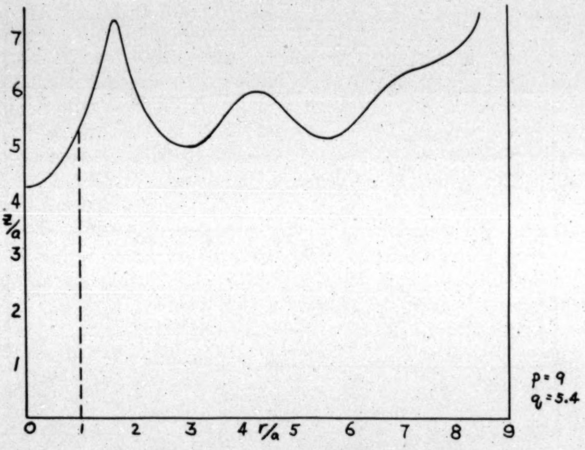


FIG. 2. Calculated electrode shape where $p = 9$, and $q = 5.4$.

It will be noted that the electrode face is not a simple curve. The hills and valleys become less pronounced for smaller values of q and in general for larger values of p. Refer also to Fig. 3 for case $p = 40$, $q = 10$. Greater values of q result in no solution being obtained for equation (19) for certain values of r. This indicates that no electrode surface satisfying the conditions required could exist at these values of r.

Keeping r constant at one of these values where normal values of z result in no solution of equation (19) and increasing z gradually from zero, shows V to increase moderately and then to decrease, passing zero and becoming increasingly negative. A solution of $V = 0$ may be obtained for certain values of r and z. However, if an electrode at ground potential would be substituted, it soon becomes obvious that the sparking distance to the high voltage electrodes would become greatly decreased. This form of solution is not practical because of the undesirably high electrical stress, even if the more difficult construction of sectional electrodes might be overcome.

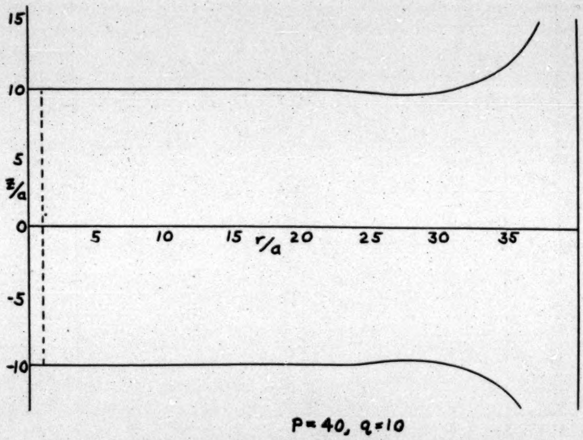


FIG. 3. Calculated electrode shape where $p = 40$, and $q = 10$.

The electrode system $V = E$ for $p = 9$ and $q = 5.4$, was tested by the use of an electrolytic tank, or trough. Advantage was taken of symmetry so that only one ungrounded electrode was used, one end of the metal trough satisfying the condition that $V = 0$ when $Z = 0$. Because of radial symmetry, only a half circumference of the electrode was required. The trough was filled with water to the horizontal diameter of the trough when the electrode was positioned symmetrically. The sides of the trough satisfied the condition that $V = 0$ when $r = pa = qa$.

In the case of this test, a was taken equal to one-half inch. A set of needle points were carefully positioned so that contact was made with the water at $r = a$ and $Z = 1/2a$, a, $3/2a$, etc., up to $5a$, with electrode at $5.4a$. Comparison was made with similar points set up at $r = -a$.

A Wheatstone bridge of the electrolytic type was set up with from three to five volts, sixty cycles per second, across the bridge. Comparison was made at one thousand cycles per second but in this case the available power was only sufficient for a rough check. In both cases the balance was determined by the use of a sensitive amplifier and a loud speaker.

Within experimental error of about one per cent., no deviation could be detected from constant gradient; which contrasts sharply with usual high voltage experience.

In order to check the advantage of using a special electrode, an experimental flat electrode of same diameter and curvature of outer edge was employed. The grounded guard cylinder and spacing for use of resistance cylinder, was identical. In this case, which is probably far better than usual in practice, there was noted an increase of potential gradient of about forty per cent., as shown in Fig. 4. It may be readily seen that the use of a flat disc for an electrode would require an overall larger size, if breakdown of resistor material or insulation is a consideration, since breakdown occurs in a region of maximum potential gradient which has been found to be at about one-third the distance down to ground from high potential end.

The elimination of capacity current in the resistor results from the distribution of potential due to the shaping of the electrode system being identical to that caused by the drop in potential along a uniform conductor.

The rather simple problem remains to select a design which would utilize these principles.

Let it be assumed that a voltmeter multiplier is to be constructed. The first consideration is to select a peak value of voltage which must not be exceeded, apply a suitable safety factor and calculate dimensions of the various parts.

The next consideration is the design of the resistor proper. With high currents a large amount of power dissipation results, which is undesirable and also requires an increase of size of both resistor and high voltage supply system. Decreasing the current means increasing the resistance and eventually this increase of resistance would entail greater cost, either for resistor or for indicating instrument.

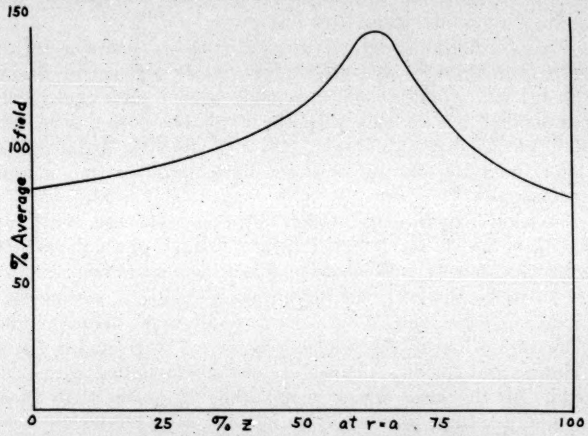


FIG. 4. Potential gradient along $r = a$ with use of flat disc.

By the use of resistance materials with controlled overall high resistivity, the high voltage resistor may be made of practically any size.

Proper selection of a unit size "*a*" will permit control of the electrode system. On the basis that the maximum peak value of the voltage is the important consideration then the appropriate radius of curvature and distance from electrode to ground alone remain to be determined. If the electrode be rounded off at 7*a* then the distance 2*a* would be the sparking distance as shown in Fig. 5.

Then for a 200 kv high voltage system, which is normally balanced 100 kv to ground, the sparking distance for 100 kv should be considered with proper safety factor.

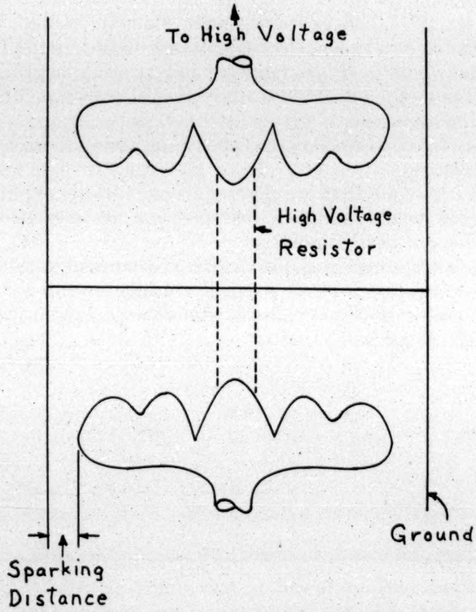


FIG. 5. Proposed electrode system for high voltage resistor.

For sphere gaps,[6] the sparking distance for 100 kv rms with 125 mm spheres at 25 degrees Centigrade and 760 mm barometric pressure is 56 mm. Then if *a* is chosen as $1\frac{1}{4}''$ then $2\frac{1}{2}''$ would be ample allowance for the sparking distance. The guard cylinder diameter would be 18*a* or $22\frac{1}{2}''$ while the distance between the backs of the electrodes might be 16*a* or 20″. The resistors would each be $2\frac{1}{2}''$ in diameter and $6\frac{3}{4}''$ in length.

---

[6] Standard Handbook for Electrical Engineers, 5th Edition, Sec. 24–2368, p. 1894.

On the same basis for a 600 kv high voltage system, balanced 300 kv to ground, the sparking distance for 300 kv rms should be considered. This is found to be 177 mm for 500 mm spheres under standard conditions so that *a* may be taken at 4″. The guard cylinder diameter would be 72″ and distance between electrode backs would be 64″. The resistors would be 8″ in diameter and 21.6″ in length.

These designs enable a rather compact voltage multiplier to be manufactured. It is not necessary to use oversize resistors to avoid breakdowns occurring, owing to high potential gradients.

It is thought that an electrode system designed on these principles would simplify the problem of insulation and measurement of high voltages.

The author expresses his thanks to a number of people for their coöperation. He is particularly indebted to Dr. C. C. Chambers of the Moore School, who served as sponsor, for his valuable suggestions.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

ABSOLUTE DETERMINATION OF THE OHM.

In a paper published in the *Journal* of the Washington Academy of Sciences for February 15, Harvey L. Curtis of the Bureau reviews the methods which have been used during nearly 100 years for the absolute determination of the unit of electrical resistance, the ohm.

Kirchhoff in 1849 laid the foundation for absolute ohm measurements by showing experimentally that a definite relationship exists between the resistivity of copper and the permeability of space. Two years later Weber used this fact in outlining the principles on which our present system of electrical units is based. Hence Kirchhoff's experiments can be used to give an absolute value of the ohm, although they were performed before our system of electrical units was conceived.

About 1860 a committee of the British Association for the Advancement of Science studied the system of electrical units proposed by Weber and decided to recommend its adoption. To make this effective, a subcommittee headed by the distinguished physicist, Clerk Maxwell, made experiments which permitted the construction of a standard resistor having a resistance of approximately 1 absolute ohm. This unit has generally been called the B. A. unit. The committee stated that their result was so accurate that never again would an absolute measurement of the ohm be necessary. The B. A. unit was the standard of resistance throughout the world for nearly a quarter of a century.

With the great increase in the commercial use of electricity which occurred about 1880, a demand arose for more accurate electrical standards. As a result, several absolute ohm determinations were undertaken. The results obtained showed that the B. A. unit differed from its intended value by 1.5

---

* Communicated by the Director.

per cent. To provide a better standard of resistance, the international ohm was established by the Chicago Electrical Congress of 1893. This unit, now almost universally employed in all resistance measurements, is the legal standard in many countries.

When the international ohm was established, absolute determinations were believed to be correct to 0.1 per cent. As no higher accuracy was needed, no absolute determinations were made for another 20 years. During this period several national laboratories were organized, and the establishment of units naturally fell to them, because of their special facilities for making precise measurements. In 1914, Smith at the National Physical Laboratory of England, showed that the International ohm differed from the absolute ohm by about 0.05 per cent. Recent investigations at the National Bureau of Standards and elsewhere have given a more accurate value for this difference, which at present is known to 0.002 per cent. When political conditions permit international gatherings, the present international ohm will probably be replaced by a standard which more nearly represents the absolute ohm.

FRICTIONAL PROPERTIES OF RUBBER.

The results of an extensive laboratory study of the coefficient of friction of rubber are reported in RP1463 by Frank L. Roth, Raymond L. Driscoll, and William L. Holt, which will be published in the *Journal of Research* for April. Most previous investigations have dealt with specific rubber products such as tires or belting, and the experimental conditions have been those under which the products are commonly used. In the present investigation it was possible to employ a relatively wide range of experimental conditions and to control the composition and surface condition of the specimens and the tracks. The principal factors considered were speed of slide, roughness of sliding surfaces, pressure of one surface on the other, size of specimens, cleanliness of sliding surfaces, and composition of the rubber.

The following results were obtained:

1. When a clean rubber specimen is towed at a constant speed of 10^{-3} cm./sec. or more along a smooth track the

friction increases, reaching a constant equilibrium value only after several centimeters of slide; correspondingly, when the specimen is allowed to slide down an inclined track under a constant load, the speed decreases to its equilibrium value. When a specimen is towed at a speed appreciably less than 10^{-3} cm./sec. the friction decreases to its equilibrium value; correspondingly, a specimen will continue to slide down an incline even after the angle of incline is decreased to a value less than the angle of repose.

2. The equilibrium coefficient of friction increases markedly with speed—the values varying from about 1 at speeds of the order of 10^{-4} cm./sec. to more than 4 at speeds of about 5 cm./sec. The occurrence of vibrations of the specimens prevented observations at speeds much over 5 cm./sec. The softer rubber compounds vibrated at even lower speeds.

3. The equilibrium coefficients of friction for very low sliding speeds are less than the coefficient of static friction, but at speeds of 10^{-3} cm./sec. or greater the dynamic coefficients are greater.

4. Except for speeds appreciably below 10^{-3} cm./sec., the rougher the sliding surfaces the lower are the observed coefficients.

5. The coefficients of friction for a rubber compound are not greatly dependent on the compounding ingredients so long as these ingredients do not bloom out on the sliding surface. The presence of materials such as bloom or talc on the surfaces decreases the values of the observed coefficients.

PROTECTION OF EYES AGAINST LIGHT FROM INCENDIARY BOMBS.

In response to inquiries from manufacturers of eye-protective glasses and from fire wardens concerning the kind and shade of tinted lens that should be recommended for wear while near an incendiary bomb, the Radiometry Section in collaboration with the Fire Resistance Section of the Bureau has made measurements on the ultraviolet emission during combustion of such bombs.

The chemical composition of an incendiary bomb is about 98 per cent. magnesium, 1 per cent. aluminum, 0.4 per cent. zinc and manganese respectively, and smaller amounts of copper, iron and silicon. The dominant color is that of burning magnesium, i.e. a brilliant white, familiar to the older photographers who used magnesium ribbon for illumination. The source of this brilliant white light is the incandescent particles of magnesium oxide which, close by the point of combustion, are at a temperature of about 2500° to 2600° C. Surrounding this ignition zone is a "fog" of magnesium oxide which intercepts and scatters radiation.

Superposed upon the continuous spectrum radiation from the incandescent particles of magnesium oxide are ultraviolet emission lines of atomic magnesium that originate at the point of combustion. Of interest in producing erythema ("ultraviolet burns") and conjunctivitis are a group of triplet emission lines at 2938A, and a 50 to 100 times more powerful resonance emission line at 2852A. Fortunately, these emission lines occur at wave-lengths that are not very effective in causing ultraviolet burns and conjunctivitis. Because of the much smaller amount of aluminum and zinc present, the ultraviolet emission lines (which are of wave-lengths shorter than 2600A) will probably not be effective in causing burns.

By means of a photoelectric ultraviolet meter at a distance of 14 ft. from the source, measurements were made on the intensity of ultraviolet radiation of wave lengths shorter than 3200A emitted by a 400 gram bomb. Filter measurements indicated that the spectral ultraviolet energy distribution was similar to that of the incandescent oxide. During the most active part of the burning of the bomb, the ultraviolet intensities of wave-lengths shorter than about 3200A varied from 11 to 15 microwatts per cm.^{2} at a distance of 3 ft. from the flame. The intensity of the ultraviolet radiation from a 1000 gram bomb would be 2 to 3 times greater. But at a distance of 3 to 4 ft. from the flame the danger from burns by ultraviolet radiation seems small in comparison with the possible injury from sparks and hot gases.

The main question, therefore, is the prevention of blurring of vision ("glare") by the intense white light emitted by the burning bomb. For this purpose dark glasses should be worn. The common colored glasses sold for sports-wear do not seem dark enough to eliminate glare. A "shade 4" tinted lens (light transmission 5 per cent.) eliminates the glare but seems too dark for the wearer to go about safely on a roof or in a darkened room. Wearing a shade 2.5 to 3 (transmission 23 and 14 per cent. respectively) will be safer in going about in poorly lighted places; and, since there is no need of standing and looking at the flame, blurring of vision (which is a temporary effect) is not likely to occur. The amount of infrared radiation from the "magnesium" and "thermite" bombs is much less than impinges on the eyes on opening the furnace door, or on staring at hot coals in an open grate fire. The various makes of blue-green, sage-green and greenish-yellow glasses (described in NBS Circular C421), shades 2.5 to 4, effectively absorb the ultraviolet and the infrared rays, and hence are to be preferred for wear near intense bright light. The light-colored blue, pink, amethyst, reddish-brown, and other nondescript colored glasses, sold for sports-wear, are not to be recommended for prevention of "glare."

A helmet with a visor that permits raising and lowering the tinted lens in front of a clear glass window, or a goggle lens with the lower half clear, seems desirable for the professional fire warden; but is probably an unnecessary expense to the average citizen.

FIRST SPECTRUM OF ANTIMONY.

Antimony in the form of sulphide, being fairly common in nature, was known to the ancients, and Oriental women are said to have used it to darken and beautify their eyebrows.

During the middle ages, compounds of antimony played a large part in the search for the elixir of life. Antimonal preparations were in great vogue as medicinal agents, and came to be so much abused that a prohibition was placed on their use by the Paris parliament in 1566.

In modern times antimony has found extensive industrial uses in type-metal alloys, antimonal lead for storage batteries, and for coverings of telephone cables. In war time it finds application in the production of ammunition, principally for the manufacture of shrapnel, shells and bullets, which require about 12 per cent. antimony in the antimonal lead.

During the past 90 years the emission spectra characteristic of antimony atoms have been described and discussed in scientific literature more than 100 times, but unfortunately it remained true that information on this subject was far from complete or satisfactory. Undoubtedly, this is attributable, at least in part, to the extraordinary character of this spectrum. When antimony atoms are excited to radiate in flames or in electrical arcs they emit almost no light in the visible range, but radiate powerfully in the ultraviolet and infrared.

William F. Meggers and Curtis J. Humphreys of the Bureau's Spectroscopy Section have completed a new study of the spectrum characterizing neutral antimony atoms. By employing efficient spectrographs and new types of photographic plates the spectrum has been photographed not only in the visible but also in the extreme ultraviolet and in the near infrared. Heretofore slightly more than 100 spectral lines were ascribed to neutral antimony. This number has now been increased to 466. The size of the largest wave-length photographed is nine times that of the shortest recorded in this spectrum. A complete list of the wave-lengths and estimated relative intensities of these antimony spectrum lines will be published in RP1464 of the *Journal of Research* for April, as a standard description of the first spectrum of antimony. In addition, these data have been carefully analyzed to see what they would reveal concerning the structure of antimony atoms. About 80 per cent. of the observed lines have been explained as atomic energy changes among 91 different states or levels. All of the atomic energy states are accounted for by 3 or 5 valence electrons, and these are precisely the antimony valences found by chemists. Several sets of energy states obeying general laws of spectral series have been found. By extrapolating such series a principal ionization potential of 8.60 volts is arrived at. This is a measure of the strength of binding of the least firmly bound valence electron in antimony atoms; it means that a free electron falling through a potential difference of 8.60 volts will acquire sufficient energy so that upon inelastic collision with an antimony atom ionization of the latter will result.

CREEP RATES OF MONEL METAL.

In the April number of the *Journal of Research* (RP1462), J. A. Bennett and D. J. McAdam, Jr., give a description of new equipment recently assembled at the Bureau for testing creep of metals. Loads up to 90,000 lb./in.^{2} can be applied on any one of twelve 0.505 inch diameter specimens held in individual furnaces, and the extension can be measured with a probable error of 0.00004 inch over the 4-inch gage length.

The apparatus was used to study the creep rate of cold drawn nickel-copper alloy over a wide range of stress and temperature. In the majority of the tests one specimen was used for a single temperature and a series of different stresses. It was found that this allowed more rapid determination of the rate which was characteristic of a given stress and temperature than if a single specimen had been used for each test. One series, run at a constant load by varying the temperature between tests, indicated that this gave a still more rapid method of determining creep rates. The data indicate that the characteristic creep rate depends only on the stress and temperature at the time of the test, and is not affected by prior stresses or temperatures. The influence of stress on the rate of creep increases with increasing stress, while the influence of temperature on the rate of creep decreases with increasing temperature. The results of the tests are shown in graphs, since no mathematical expression was found which would represent them.

SOIL CORROSION OF FERROUS AND NONFERROUS METALS.

Since the inauguration of the Bureau's field tests of the corrosion of metals in soils, laboratory investigations have been conducted in order to develop methods for identifying corrosive soils and to study corrosion processes under the soil conditions represented in the field tests. Because ferrous metals are widely used underground, it is natural that the first series of studies should have been restricted to the corrosion of steel. Recently the laboratory tests have been extended to include materials other than low-carbon steel, namely, the stainless steels, copper, low-copper brass, lead, and zinc. A report of this investigation by I. A. Denison, entitled "Electrolytic Behavior of Ferrous and Nonferrous Metals in Soil-Corrosion Circuits," will be published in the *Transactions of the Electrochemical Society*. Preprints will be available about April 15.

In this investigation, a specially designed corrosion cell, previously used in measuring the corrosion rate of steel, was employed. With the more corrodible metals, such as steel and zinc, and other metals in certain environments, the corrosion cell develops its own e.m.f. To measure the corrosion rate of these metals for comparison with the results of field tests, the electrodes of the cell were brought to the same potential by the application of an external electromotive force, the corresponding cell current being taken as proportional to the corrosion rate. For all other materials and environments in which the cell did not develop a definite polarity, the minimum current required to protect the cathode from corrosion was taken as equal to the corrosion current. Data are presented which show that when the rate of corrosion is controlled at the cathode, the current required to bring the electrodes of the cell to the same potential is equal to the protective current.

The rates of corrosion of metals in different soils were generally in agreement with the results of the field tests in that both series of tests indicated whether or not the materials were seriously corroded in the different environments. Zinc, however, which was corroded more rapidly than steel in the laboratory, was more resistant than steel in the field tests. The relatively greater corrosion resistance shown by zinc in the field tests is attributed to the partial suppression of anodic areas by cathodically formed alkali. The corrosion rates obtained with the corrosion cell might be taken to indicate the behavior of zinc as a separate anode in a bimetallic circuit, as in the cathodic protection of steel by zinc.

The electrode at which the corrosion rate is controlled was determined for the different materials and environments by observing current-potential curves of the separate electrodes. The rates of corrosion of steel and zinc under most soil conditions are controlled by the reaction at the cathode. However, in very porous soils, particularly those strongly alkaline in reaction, the corrosion rate is controlled anodically. The corrosion of copper, brass, and stainless steel, which in most soils is controlled at the anode, is controlled by the cathode in poorly aerated organic soils containing high concentrations of soluble salts, indicating that the normally protective oxide film has been destroyed. For lead, however, the shift of control is in the opposite direction, becoming anodic in the presence of soluble salts because of the formation of a protective film of corrosion products at the anode.

High Power Testing Pushed to New Limits.—W. B. BATTEN (*Power Plant Engineering*, Vol. 46, No. 3.) Facilities are now available for testing short circuits of 2,000,000 kv.-a. in the new high power laboratory at the Westinghouse East Pittsburgh Works. In addition, the previous current limit of 3200 amp. on a 132 kv. 3 phase circuit has been extended to 7500 amp. Test voltages have been raised from 230 kv. to 345 kv. 3 phase and from 264 to 396 kv. single phase with mid-point grounded, thus amply exceeding present transmission voltages. The laboratory is in effect a power station delivering a short circuit output comparable with that on large power systems. In contrast it carries no continuous load, and a short circuit is a normal instead of an abnormal condition. The flexibility of the station permits duplicating of many voltages and currents corresponding to different services, classes, and ratings. Accurate control of service conditions and complete records of performance produce experimental data which, when properly used, result in new designs of proven performance. Highlighting the testing equipment in the new laboratory are two 60,000 kv.-a. generator sets. These machines when operating in parallel can produce an initial thru phase symmetrical short circuit of 2,200,000 kv.-a. at 13.2 kv. The stators are spring mounted to cushion the shock on the foundation at the time of short circuit. The stored energy in the rotors of these sets is adequate to absorb the losses produced by short circuits and it is unnecessary to disconnect the driving motors except in special cases where the short circuit is sustained for several seconds. This stored energy presents a problem in bringing the sets to a stop. With no braking, it takes two hours for the set to come to a standstill. Second in importance is a transformer station that includes six 33,333 kv.-a. single phase transformers. Other features of the laboratory are high current transformers to provide momentary and 5-second current-carrying tests, a refrigerated room for testing breakers at minus 20 degrees F., and the necessary complement of control and measuring equipment.

R. H. O.

THE FRANKLIN INSTITUTE.

MEDAL DAY MEETING,

WEDNESDAY APRIL, 15, 1942.

This year, to meet varying conditions, the annual Medal Day meeting was held in April instead of May.

The meeting was called to order at three-thirty o'clock by the President, Mr. Charles S. Redding.

The President announced this to be a regular stated monthly meeting and asked if there were any objection to the minutes of the previous month, which had been printed in full in the April JOURNAL. No objection being offered, the minutes were declared approved as printed.

Dr. Henry Butler Allen, Secretary and Director of the Institute, was then called upon for a report. He stated that routine business was omitted in deference to the importance of awarding medals.

Mr. Redding then outlined the procedure followed by the Committee on Science and the Arts, the body within the Institute responsible for recommending award of medals. Its recommendations are made to the Board of Managers, which group gives the final vote to the award. After commenting on the fact that the Committee has been functioning since the inception of The Franklin Institute in 1824, he proceeded with the business of the day, which embraced the award of the following medals:

	Sponsors
Presentation of Certificates of Merit THE GOODYEAR TIRE AND RUBBER COMPANY Akron, Ohio	
Received by PAUL W. LITCHFIELD, Chairman of the Board, Goodyear Tire and Rubber Company,	Dr. J. W. Hill
JOHN J. GREBE, D.Sc., Director, Physical Research Laboratory, Dow Chemical Company, Midland, Michigan.	Mr. E. L. Forstall
WALTER LARKIN, Fidelity Machine Company, Philadelphia, Pennsylvania.	Mr. J. S. Rogers
Presentation of Longstreth Medals Jointly to	
RALPH E. FLANDERS, D.Sc., D.ENG., President, and ERNEST V. FLANDERS, Manager, Thread Grinder Division,	Mr. F. G. Tatnall

Jones and Lamson Machine Company,
Springfield, Vermont.

Charles Maxwell Kearns,
Dr. R. Eksergian
Project Engineer,
Hamilton Standard Propeller Company,
East Hartford, Connecticut.

Presentation of Brown Medal
Duff Andrew Abrams,
Professor C. M. Gay
Consulting Engineer,
New York City.

Presentation of Levy Medal
John Donovan Strong, Ph.D.,
Mr. J. Winlock
Assistant Professor of Physics in Astrophysics,
California Institute of Technology,
Pasadena, California.

Presentation of Potts Medals
Jesse Wakefield Beams, Ph.D.,
Dr. W. R. Wright
Professor of Physics,
University of Virginia,
Charlottesville, Virginia.

Harcourt Colborne Drake,
Dr. W. S. Pardoe
Director of Research,
Sperry Products, Inc.,
Hoboken, New Jersey.

Bernard Lyot, D.Sc.,
Mr. W. H. Fulweiler
Meudon Observatory,
Paris, France,
Received by M. **Patrick Coppinger**, Consul for France.

Presentation of Cresson Medals
Claude Silbert Hudson, Ph.D.,
Dr. J. W. Hill
Professor of Chemistry,
National Institute of Health,
U. S. Public Health Service,
Bethesda, Maryland.

Isidor Isaac Rabi, Ph.D.,
Dr. C. B. Bazzoni
Professor of Physics,
Columbia University,
New York City.

Presentation of the Franklin Medal and Certificate of Honorary Membership
Jerome Clarke Hunsaker, D.Sc.,
Dr. Frederic Palmer, Jr.
Professor in Charge,

Daniel Guggenheim Aeronautical Laboratory,
Massachusetts Institute of Technology,
Chairman, National Advisory Committee for
Aeronautics.

Presentation of the Franklin Medal and Certificate of
Honorary Membership
PAUL DYER MERICA, PH.D., Sc.D., Dr. Frederic Palmer, Jr.
Vice-President and Director,
International Nickel Company of Canada.

PAPERS.

"Research in Aeronautics"
by
DR. JEROME C. HUNSAKER

"The Precipitation—Hardening of Metals"
by
DR. PAUL D. MERICA

The meeting was adjourned with an expression of appreciation to the Medalists for their coöperation in taking time out of their busy lives to be present in person to receive their awards; and with a word of welcome to Medalists of former years who attended this annual function.

All guests were invited to visit the museum and the library upon their departure from the lecture hall.

(A detailed statement of the afternoon and evening events of Medal Day, together with papers presented, will appear in an early issue of the JOURNAL.)

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from two until ten o'clock P.M.

RECENT ADDITIONS.

AERONAUTICS.

DOANE, F. B., AND W. E. THOMAS. Magnaflux Aircraft Inspection Manual. Second Edition. 1941.

SEARS, WILLIAM R. The Airplane and Its Components. 1942.

BIBLIOGRAPHY.

FAŸ, BERNARD. Bibliographie Critique des Ouvrages Français Relatifs aux États-Unis (1770–1800). 1925.

BIOCHEMISTRY.

TAYLOR, F. SHERWOOD. The Conquest of Bacteria. 1942.

BIOGRAPHY.

JOHNS, BESSIE W. Poor Richard Comes to Life. 1942.

SWIFT, LINDSAY. Benjamin Franklin. 1910.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

KRAEMER, ELMER O., Editor. Advances in Colloid Science. Volume 1. 1942.

LEWIS, WARREN K., LOMBARD SQUIRES, AND GEOFFREY BROUGHTON. Industrial Chemistry of Colloidal Amorphous Materials. 1942.

Nord, F. F., AND C. H. WERKMAN, Editors. Advances in Enzymology. Volume 2. 1942.

PALMER, W. G. Experimental Physical Chemistry. 1941.

PEASE, ROBERT N. Equilibrium and Kinetics of Gas Reactions. 1942.

TAYLOR, HUGH S., AND SAMUEL GLASSTONE. A Treatist on Physical Chemistry. Third Edition. Volume One. 1942.

EDUCATION.

Benjamin Franklin's Proposals for the Education of Youth in Pennsylvania 1749. 1927.

ELECTRICAL ENGINEERING.

HARTSHORN, L. Radio-Frequency Measurements by Bridge and Resonance Methods. 1941.

KRAEHENBUEHL, JOHN O. Electrical Illumination. 1942.

ENGINEERING.

RUBEY, HARRY, GEORGE EDWARD LOMMEL AND MARION WESLEY TODD. Engineering Surveys: Elementary and Applied. 1942.

U. S. BUREAU OF STANDARDS. Standardization Activities of National Technical and Trade Organizations by Robert A. Martino. 1941.

VAN DEN BROEK, J. A. Elastic Energy Theory. Second Edition. 1942.

GRAPHIC ARTS.

BLADES, WILLIAM. The Pentateuch of Printing. 1891.

DONNELLY & SONS COMPANY, R. R. A Rod for the Back of the Binder. 1928.

Fleuron: a Journal of Typography. Edited by Oliver Simon. Numbers 1–3. 1923–1924.

MCMURTRIE, DOUGLAS C. American Type Design in the Twentieth Century. 1924.

MCMURTRIE, DOUGLAS C. Printing Plant of the Oxford University Press. 1923.

MORISON, STANLEY, Editor. On Type Faces—Published Jointly by The Medici Society—and The Fleuron. 1923.

Moxon's Mechanick Exercises or the Doctrine of Handy-Works Applied to Printing. A Literal Reprint in Two Volumes of the First Edition Published in the Year 1683. With Preface and Notes by Theo. L. DeVinne. 1896.

TREDWELL, DANIEL M. A Monograph on Privately Illustrated Books. 1892.

HISTORY.

Adams, James Truslow, and Charles Garrett Vannest. The Record of America. 1935.

Fay, Bernard. L'Esprit Révolutionnaire en France et aux États-Unis à la Fin du xviii^{e} Siècle. 1925.

INDUSTRIAL MANAGEMENT.

Barnes, Ralph M. Motion and Time Study. Second Edition. 1940.

Buckingham, Earle. Production Engineering. 1942.

Bullinger, Clarence E. Engineering Economic Analysis. First Edition. 1942.

MANUFACTURES.

American Foundrymen's Association. Transactions. Volume 49. 1941.

Witham, G. S. Modern Pulp and Paper Making. Second Edition. 1942.

MECHANICAL ENGINEERING.

Axelrod, Aaron. Machine Shop Mathematics. 1942.

Begeman, Myron L. Manufacturing Processes. 1942.

MILITARY ART AND SCIENCE.

Armstrong, Hamilton Fish. Chronology of Failure. 1941.

Dean, Vera Micheles. Europe in Retreat. Third Edition. 1941.

Farago, Ladislas, Editor. The Axis Grand Strategy. 1942.

Foertsch, Hermann. The Art of Modern Warfare. 1940.

Root, Ralph Rodney. Camouflage with Planting. 1942.

"Steel." Modern Shell Production. 1941.

NAVAL ART AND SCIENCE.

Mahan, A. T. Sea Power in its Relations to the War of 1812. In Two Volumes. 1905.

Tellier, Albert J. Tellier's Navigator's Compendium. Fifth Edition. 1941.

PATENTS.

United States Patent Office. Index of Patents Issued 1941. 1942.

United States Patent Office. Index of Trade-Marks 1941. 1942.

PHILOSOPHY.

Stace, W. T. Destiny of Western Man. 1942.

PHYSICS.

Duff, A. Wilmer, and Morton Masius. College Physics. 1941.

King, Horace W., Chester O. Wisler and James G. Woodburn. Hydraulics. Fourth Edition. 1941.

Synge, J. L., and B. A. Griffith. Principles of Mechanics. First Edition. 1942.

SANITARY ENGINEERING.

Eldridge, E. F. Industrial Waste Treatment Practice. First Edition. 1942.

NOTES FROM THE BARTOL RESEARCH FOUNDATION.

THE BEHAVIOR OF PROPORTIONAL COUNTER AMPLIFICATION AT LOW VOLTAGES.*

BY

M. E. ROSE AND W. E. RAMSEY.

In a recent paper [1] an investigation of the general character of gas amplification in proportional counters was described. This work was confined to the measurement of high amplification factors as a function of counter voltage, counter geometry, nature of gas filler, and gas pressure. Instrumental limitations prevented the pursuit of these studies into the region of low gas amplification where the transition to ionization chamber behavior is to be expected. In order to explore this region and to verify the essential correctness of the ideas previously presented the measurements have been extended by a radically different method. The principle of the method is based on the comparison of currents in a counter measured at potentials above and below the threshold voltage (voltage at which gas amplification factor begins to depart from unity). The amplification factor at any voltage is then directly obtained as the ratio of these currents.

A tube has been designed which serves equally well for measuring ionization currents in the chamber with and without gas amplification. While the customary counter geometry was preserved all the constructional refinements necessary for the measurement of small ionization currents were incorporated. Since the smallest currents involved are less than 10^{-14} ampere, the need for this care is apparent. A description of the apparatus is given in the caption to Fig. 1(A).

In Fig. 1(B) the amplification factor as a function of counter voltage is given for an argon-oxygen mixture and for two pressures of commercial methane. The voltage region over which the tube is operating as an ionization chamber is immediately evident from these curves. While a

---

* Reprinted from *The Physical Review*, **61**, 198 (1942).

[1] M. E. Rose and S. A. Korff, *Phys. Rev.*, **59**, 850 (1941).

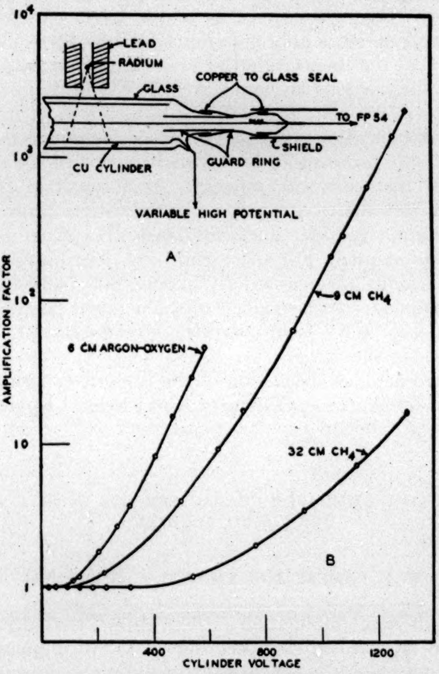


FIG. 1. A. One end (other of similar construction) of the ionization chamber-counter. The central wire connected to the floating grid of FP54 electrometer tube, grid current 10^{-15} amp. Grid of tube maintained at constant potential by compensating inductively for flow of charge to the wire. These known induced potentials provide a measurement of the charge on the wire. Range of measurement extended over wide limits by adding capacity to wire system. Shield and holder for FP54 tube evacuated to eliminate spurious ionization currents. Elimination of leakage currents by guard rings demonstrated by complete run with radium source removed. B. Amplification factor (logarithmic scale) against cylinder voltage for 6-cm. argon-oxygen ($0.94A$-$0.06O_2$) and two pressures of CH_4.

threshold voltage is apparent in the present results, this is not to be identified with the threshold voltage as defined in the previous work.[1] According to the theory there developed [cf. Eq. (14)], the amplification curves should show a sharp cut-off at voltages considerably above those observed here. This disagreement is only apparent and is to be explained by the fact that the theory referred to is based on a model which neglects fluctuations in ionization and energy loss of the secondary electrons. These fluctuations are most important at low amplification and the observed behavior is readily interpreted in terms of an energy distribution of the electrons arising in the avalanche process. As a result of the finite width of this distribution the multiplication process, at a given counter voltage, does not start at a fixed point, as previously assumed, but there will be a distribution of such points. Finally, the essentially exponential behavior of the amplification factor, as evidenced by the linear portions of the curves in Fig. 1(B), is in complete agreement with the prediction of the theory.

A more detailed discussion of the present experiment will appear in the April, 1942 issue of The Physical Review.

SOME DISCHARGE CHARACTERISTICS OF SELF-QUENCHING COUNTERS.*

BY

W. E. RAMSEY AND EMMETT L. HUDSPETH.

In a previous publication [1] it was shown that the counter discharge mechanism proposed by C. G. Montgomery and D. D. Montgomery [2] satisfactorily explains the dependence of pulse size upon counter wire capacity and counter length for a non-self-quenching gas mixture. Similar studies made with a self-quenching gas provide equally satisfactory agreement and illustrate nicely the essential difference between the two types of counter operation. As anticipated the self-quenching mixture provides no mechanism for insuring a constant pulse size (for varying capacity and length) and the total voltage swing of the wire for a given cylinder potential is simply $l\alpha/c$. Here l is the effective counter length, c is the total capacity of the wire system and α is the charge per unit length in the positive ion sheath which surrounds the counter wire during the discharge.

---

* Reprinted from *The Physical Review*, **61**, 95 (1942).

[1] W. E. Ramsey and Wayne L. Lees, *Phys. Rev.*, **60**, 411 (1941).

[2] C. G. Montgomery and D. D. Montgomery, *Phys. Rev.*, **57**, 1030 (1940).

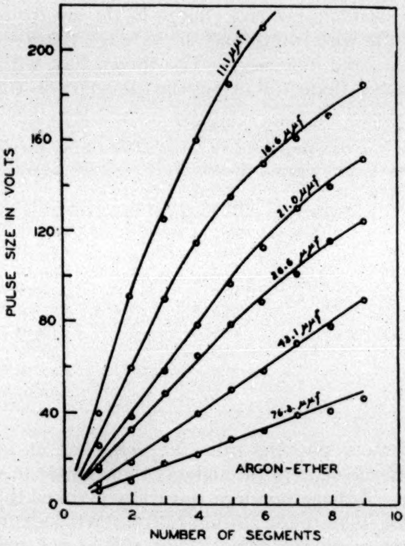


FIG. 1. Pulse size *vs.* length and capacity of counter.

Figure 1 displays a family of curves showing the dependence of pulse size upon length and capacity while Table I gives the values of α as calculated from the slopes of the separate curves at $l = 0$. For small c and large l a departure from the simple relation indicated is observed and corresponds to that found in the non-self-quenching mixture where c is small and l is large. It has been previously shown [1] that the voltage pulse can be represented best by the expression $(l/c)[\alpha - (\beta l/c)]$, where $\beta l/c$ is negligible except for a long counter with a small total wire capacity. For both types of gases the departure is to be expected if a time of the order of magnitude of 10^{-7} second is required for the discharge to spread the entire length of the tube. Such a time delay permits the potential of the wire to change while the positive ion sheath is being established in certain portions of the counter. For this reason the positive charge in the sheath is not constant over the wire length and the average value decreases as we decrease c and increase l. The theory has, with justification, assumed that, following the passage of the original ionizing particle, photons from the first electron avalanches create discharge centers throughout the counter in extremely short times. It has, therefore, naturally invoked the assumption that, for its purpose, the ionization starts instantaneously at all points along the wire. That this is not strictly true follows from the fact that the spreading process depends upon electrons moving appreciable distances through the gas. Therefore the absence, rather than the appearance, of a small correction term would occasion some surprise.

TABLE I.

Values of α, the Charge per Unit Length in the Positive Ion Sheath.

Capacity $\mu\mu f$	$\frac{\text{Volts}}{\text{Segment}}$	α in coulombs
11.1	40	4.4×10^{-10}
16.6	26	4.3×10^{-10}
21.0	19.5	4.1×10^{-10}
26.6	16.2	4.3×10^{-10}
32.1	13.0	4.2×10^{-10}
43.1	10.0	4.3×10^{-10}
54.2	8.2	4.4×10^{-10}
65.2	6.5	4.2×10^{-10}
76.3	5.5	4.2×10^{-10}

Figure 2 of reference 1 is a diagram of the counter used. In order to vary the effective counter length *without* changing the wire capacity the cylinder is made to consist of nine separately insulated segments whose voltages may be independently assigned. The number of segments at a fixed value above the starting potential determines the effective counter length. All other segments are at a given voltage well below the Geiger counter threshold. The total capacity of the wire system was varied by adding capacitors between wire and ground. The pulses were observed on an electron tube oscillograph under conditions which gave more than adequate time for the complete collection of the charge involved in the discharge process.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Seminar on Physiological Processes Involved in Wound Healing.—DR. LESLIE B. AREY, Head of the Department of Anatomy of Northwestern University Medical School, discussed the physiological processes involved in wound healing before the Seminar group of the Biochemical Research Foundation on March 31. The subject of healing is an extremely extensive one, with many ramifications.

The general phenomenon of healing differs in man and lower vertebrates, the latter having much greater capacity for healing than man. Muscle does not regenerate; a wound in muscle makes a fill of connective tissue to bring the parts together. Peripheral nerves make a good repair if the injury is not too great, and epithelial and connective tissue make excellent repair.

It is customary to differentiate healing into two types, primary and secondary repair (or intention). Primary wound healing is concerned with a superficial or cleft-like wound which may be healed by simple epithelial regeneration, direct union of adherent surfaces, or regeneration under a scab. Secondary repair takes place when the wound is an open one involving gross defects. A fill of new tissue must replace the part which has been torn away or destroyed. This healing by so-called granulation tissue may also take place under a scab.

The events of wound healing are concerned with a provisional closure and subsequent destructive and constructive changes which remove the dead tissues and replace them with new cells. In general if the wound extends through the epidermis the tissues are separated, dislocated, and partially destroyed. The first thing which occurs is a flow of blood which may stop spontaneously even in rather large vessels. In great injuries, for example when a limb is torn off, there is often very little bleeding. It is certain that thrombosis occurs to some extent, but probably segmental spasms in the vessels, or the tearing away of the lining forming a rosette-like arrangement, also account for spontaneous cessation of bleeding. Torn capillaries studied under a microscope show no thrombosis.

Traumatic inflammation always accompanies a wound. There will also be found hyperemia, exudation, and leucocytic emigration. These lead to the formation of a provisional fibrin cover which seals the edges of the wound, to the washing away of noxious substances, the resorption of blood and other exudates, and the removal of tissue fragments by cellular activity. Clot formation closes a wound, sometimes in a few minutes, to stop the loss of fluid and seal the edges together. This clot is transformed by syneresis from a jelly-like mass to an impermeable scab which offers a substrate for invading new cells.

Three zones are involved in the destructive changes which take place during healing. The first zone is made up of the cells bordering the edge of the wound; these cells are fatally injured. The next layer has injured traumatized cells which also usually die. The third layer appears normal but contains many cells which have been injured fatally. The first two zones are especially open to infection. The death of the tissue may be due directly to traumatization or indirectly to metabolic deficiencies such as injured blood supply or lymphatics. The resulting destruction is either by autolysis, in which the cells elaborate pepsin- and trypsin-like enzymes, or by heterolysis in which the enzymic action is through leucocytic activity and mostly in the necrotic area. The neutrophils produce leucotryptase, which attacks dead cells and altered connective tissue fibers, and helps remove fibrinous deposits.

The impaired blood supply leads to tissue hunger, which in turn induces cell division. The products of cell destruction are also stimulative, growth being most abundant about the site of focus of cell injury.

The first fluid to be secreted by a wound is unmodified blood and lymph; the amount is reduced as fibrin deposits and covers the wound. The composition also changes; plasma constituents, especially fibrinogen and globulin, are increased as long as the wound is open, but healthy granulation tissue has scanty secretion and relatively few leucocytes.

Infection controls the history of the healing, some even claiming that infection in other parts of the body affects the course of the wound healing.

The constructive phase of wound healing frequently overlaps the destructive phase just described; a relapse may cause reversion to destruction. Connective tissue participation involves normal connective tissue cells plus some derivatives resulting from inflammation. The macrophages and fibroblasts have important roles. In primary healing the proliferation becomes predominant by the second day. In secondary healing there is a definite latent period, the wound beginning to fill with connective tissue in six to eight days. This tissue is called "granulation" tissue because the vascular cores surrounded by young fibroblasts and wandering cells give it a pebbly appearance. Closely opposed skin wounds in man and swine lack granulation tissue because of the amount of fat in the skin layers; this encroaches on the wound area and furnishes a good substrate for migratory and reparative cells. Granulation tissue provides a cellular protective membrane which is less permeable than the fibrin membrane and is an excellent defense against infection. In aseptic, apposed wounds, fibroplasia appears in a few days; in open wounds healing by granulation the production of fibers appears when the destructive phase is ended and the fibroblasts mature. The earliest fibrils are sparse, delicate, and rich in water. The severed fibers are united by a splice of new fibers, aided by swelling and solution of old fibers. There is a question whether these new fibers are elaborated in the protoplasmic cell or are formed by the action of enzymes in the structureless gels which lie between the cells. The most authoritative names favor the second interpretation. With the formation of fibers scar tissue is organized. The tensile strength of the scar tissue can be tested, and the maximum healing occurs in about 14 days. Young scars gradually become less vascular, less cellular, and paler. It is sometimes a question of months before elastic fibers make their appearance.

Epithelial participation in wound healing is concerned with the filling of the wound by cells from the lower layer. Filling may occur in a day if there is good apposition to the edges of the wound; a surgical wound requires longer to fill, but cellular migration starts within one hour. Cells spread over and closed very small wounds in fish or amphibia in 2 or 3 hours. The granular and horny layers of the skin produce a syncytial mass with rod-shaped nuclei which covers the scab, invades, and gradually dissolves it. The malpighian layer becomes elongate cells which form a syncytium both invading the scab and passing beneath it. This lowest layer remains when the scab falls off. Just how the epithelial and connective tissue layers are reunited is not established.

Cellular migration is the primary factor involved in the mechanism of wound closure. If enough cells are available amebism closes a wound; an injury inflicted on the cornea, for example, is restored in a very short time. Proliferation is usually a later phenomenon, but is highly important in the closure of large wounds. Contraction also plays an important role in closure of medium-sized and large wounds of loose skin, entering after several days and continuing until epithelial edges are 10–15 mm. apart. It is negligible in wounds smaller than 10 mm.

The healing pattern in wounds has been studied very thoroughly, and is characteristic in wounds of moderate size. There is a latent period of three or four days with no change in size of the wound. When contraction starts, it is active at first, then slows down, the rate being proportionate to the size of the wound. The rate of epithelization is fastest as healing nears completion, but overlaps the contraction stage. Formulae for predicting wound healing have been worked out by De Noüy and others. The progress and rate of healing have also been measured by testing the tensile strength.

There are many physico-chemical phenomena connected with wound healing. Fluid content of wound tissue increases within three hours after wounding and reaches a maximum at one day. The tissues are more sol-like. Electrolytes, potassium at least, are increased; there is a greater depression of the freezing point. There is an electric potential with respect to the nearby skin, these potentials being related to fibroplasia. The pH of the fluid is pushed toward the acid side, especially in the early stages. This favors enzyme autolysis and phagocytic activity. It produces capillary dilation, thereby slowing the flow. It favors the transmission of plasma colloids and the swelling of tissue colloids, and is one of the things related to pain.

The factors influencing healing constitute a very controversial subject. Age is an important factor and is included in the formulae for predicting healing because it is an indication of physiological activity. Fibroplasia begins earlier in the young, and is less inhibited. Mild pressure is said to be favorable. Rise in temperature accelerates healing in warm-blooded animals, and in cold-blooded animals the healing follows the coefficient of doubling for every ten degrees rise. Sympathectomies produce a rise in temperature and more rapid healing. Ultraviolet light in mild doses is accelerating, as are also roentgen rays in small doses, but large doses retard healing. The effect of these latter is on the connective tissue rather than on the epithelium. An acid diet favors clean and rapid healing, the effect being to shorten the latent period. The efficacy of vitamins and hormones is a much-disputed subject, but vitamin C has been shown to be definitely beneficial; a deficiency produces a delayed or incomplete healing because collagen does not form. In scorbutic patients scars soften and wounds disrupt. Dr. Crandon of Boston made himself scorbutic, and had wounds inflicted and biopsied during the course of the experiment. At three months wounds healed perfectly, but at six months he had a bad scorbutic condition and, although wounds seemed to heal, the stitches dropped out, and biopsies showed that healing was entirely epithelial. Ascorbic acid administration for ten days led to prompt healing of incisions. The literature on the subject of hormonal effects on healing is a little too confusing to allow any final deductions to be drawn. Every ductless gland in the body has been attributed with the power to promote healing. There is also much indecision about the efficacy of tannic acid, gentian violet and codliver oil, and some investigators suggest that certain antiseptics may even retard healing. Allantoin and urea are generally credited with accelerating healing.

A comprehensive review and bibliography were published by Dr. Arey in Volume 16 of *Physiological Reviews* (1936). (Reported by RUTH E. CORBET.)

BOOK REVIEWS.

THE CONDENSED CHEMICAL DICTIONARY, by The Editorial Staff of the Chemical Engineering Catalog. Third Edition, completely revised and enlarged under supervision of Thomas C. Gregory, Editor. 756 pages, 16 × 23 cms. New York, Reinhold Publishing Corporation, 1942. Price $12.00.

The many users of the Condensed Chemical Dictionary will be interested to learn that the third edition has now been published. The general arrangement remains in the same handy fashion as the previous editions but over 6000 new items have been added. They comprise chemicals, drugs, pharmaceuticals, chemical specialties, metals, minerals, clay products, petroleum products, essential oils, perfumery chemicals, leather processing agents, mothproofing agents, insecticides, fungicides, hydrogenated products, flavoring materials and others. There is also included Typical Specifications secured from manufacturers. They represent a description of the product as marketed and sold commercially.

This book is just crammed full of useful, valuable information to all those engaged in chemical or related activities—chemists and scientists as well as non-technical people. It is a foremost reference work.

R. H. OPPERMANN.

COLLEGE PHYSICS, by William T. McNiff. 657 pages, illustrations, 16 × 24 cms. New York, Fordham University Press, 1942. Price $4.00.

Because textbooks on College Physics are numerous it might lead to the conclusion that the physics teacher has little difficulty in selecting just what he needs. On the contrary, the large number of texts available has enlarged the difficulty rather than minimized it. Almost every text is of a different shade, and many for a slightly different niche. Some are of the sugar-coated variety designed to make physics easy either by direct sacrifices of important material or insufficient rigorousness. To the other extreme there is the straight-from-the-shoulder type, dry, accurate, boring, and little recognizing that the student is human. A conservative stand would not be midway between the two extremes, but would be partial toward the latter, yet retaining those things which stimulate interest. The book at hand falls into this class.

This third edition combines into one volume the two volumes of each of the previous editions. The author clearly recognizes the necessity of teaching the coordination and interrelation of physics and mathematics and directly pursues this course. The first general division is into what is termed book one and book two. Book one covers mechanics, heat and sound. Under mechanics, at the outset, there is a change from the usual sequence. After treating on the properties of matter and physical units of measurement, the mechanics of fluids is taken up, gaining the advantage of an easier approach to the more difficult mathematical divisions of the subject matter. Following this there is the physiological and therapeutical effects of changes in air density. This provides an informative stimulus to further work which is carried through motion, force, acceleration friction, machines, and work and energy. The sequence through the study of heat is not much different from other texts, and sound is similar.

Book two is the larger part of the work. It gives a good conception of the character of light, its reflection and refraction. Spherical mirrors are taken up followed by lenses and optical instruments which leads to spectrum analysis and polarization. The study of magnetism is followed by a rather complete coverage of electricity. Here the elements of electricity are given attention as well as generators and motors, conduction of electricity through gases, radio activity, and radio telegraphy, telephony, television and the electron microscope. The last part of this book is devoted to modern physics where modern ideas are introduced.

The author has given a great deal of attention to the simpleness of treatment, accomplishing much in preventing mysteries which may arise in the mind of the student. Illustrative, worked out problems are given which assist greatly in understanding the subject matter. Another feature worthy of note is the size and style of type in which the book is printed. It adds greatly to ease in reading and the order of vision in the mind of the subject matter.

R. H. OPPERMANN.

SCIENCE CALLS TO YOUTH, by Raymond F. Yates. 205 pages, illustrations, 14 × 20 cms. New York. D. Appleton-Century Company, 1941. Price $2.00.

In this, his most recent contribution to the field of popular science, the author explains, clearly and simply to the youth of our nation—boys and girls alike—what they may accomplish in the field of science when someday they take their places in the world of affairs. The great value of this volume lies in its vocational guidance possibilities.

Once more is sounded the emphatic note of how those, in whose hands are the reins of the nation's business, are calling more and more frequently upon science to solve business problems. Raymond Yates tells of the scientists who have pioneered as executives in the world of industry. He plainly depicts the rich opportunities awaiting the scientists of tomorrow in many fields of endeavor.

Another relatively important aspect of this work is that it is written in terms readily understood by the lay reader, whose academic background has not included formal scientific training. Abundant and well selected photographic illustrations play their important role in fostering a vigorous curiosity in the minds of young readers. Excerpts of the "success story" type add their weight to the establishing of balanced concepts so necessary for our youth in their evaluation of their own potentialities as related to the work of the world. Science no longer seems a mysterious activity of the laboratory. Scientists no longer appear as men apart, engaged in matters far removed from a breathing world.

J. HUBLEY SCHALL, 3D.

PUBLICATIONS RECEIVED.

The High-Speed Compression-Ignition Engine, by C. B. Dicksee. 331 pages, illustrations, 16×22 cms. New York, Interscience Publishers, Inc., 1940. Price $4.50.

Advances in Colloid Science, edited by Elmer O. Kraemer, Ph.D., Volume one, 434 pages, illustrations, 16×23 cms. New York, Interscience Publishers, Inc., 1942. Price $5.50.

The High-Speed Internal-Combustion Engine, by Harry R. Ricardo, F. R. S. Revised by H. S. Glyde, B.Sc. (Eng.) London. 434 pages, illustrations, 18×25 cms. New York, Interscience Publishers, Inc., 1941. Price $7.50.

Electric Circuits and Machinery, Volume II, Alternating Currents, by Frederick W. Hehre and George T. Harness. 635 pages, illustrations, 16×24 cms. New York, John Wiley & Sons, Inc., 1942. Price $6.00.

The Physical Examination of Metals, by Bruce Chalmers and A. G. Quarrell, Volume II, Electrical Methods, 280 pages, illustrations, 15×22 cms. New York, Longmans, Green & Co., 1942. Price $6.00.

The Electron Microscope, by E. F. Burton and W. H. Kohl. 233 pages, illustrations, 16×24 cms. New York, Reinhold Publishing Corporation, 1942. Price $3.85.

CURRENT TOPICS.

A Stove Manufacturer Builds Lifeboats.—(*Iron Age*, Vol. 149, No. 11.) Away out in Kokomo, Indiana, the Globe American Corp., a stove manufacturer, found itself faced with the prospect of either getting war contracts or going out of business. Through the farsightedness of the executives of the 50-year-old concern, a changeover was made in plant facilities to make steel lifeboats, with which every oceangoing vessel is equipped, and in spite of the fact that the plant is 1000 miles from the Atlantic Ocean and 2000 miles from the Pacific, the U. S. Maritime Commission considered the design submitted by Globe American as fine enough to be adopted as "standard equipment" for the first 312 Liberty cargo ships now being built. Previously, lifeboat design was not standardized, but had only to meet certain minimum specifications pertaining to size, material, air tank capacity, etc., as established by the Bureau of Marine Inspection and Navigation. Globe American, anxious to get into war work, employed naval architects to design a new, improved steel boat, and through their pioneering efforts obtained a contract for 1248 such boats. They will be identical in every respect and are constructed at the rate of one every 2 hr. The three plants are now working one eight hour shift a day. The rate of production can be increased 150 per cent. by working 24 hr. a day, high enough to provide enough lifeboat equipment for the launching of nine cargo vessels a day. In the shop, the boats are moved by a power conveyor along the production line from the time the keel is laid until the finished craft is loaded into the railroad car. The construction is entirely of steel except for wood floor boards, seats, mast, oars, and rudder, and when shipped the boats are completely equipped with clocks, releasing gear, oars, red sails, food and water, signaling and first aid equipment, canvas covers, hood and weather cloths. There are four boats to each cargo ship, one of which is equipped with a built-in marine motor. Each boat seats 31 people, providing twice the capacity of a maximum crew of each vessel.

R. H. O.

World's Longest Underwater Natural Gas Pipeline.—(*Manufacturers Record*, Vol. 111, No. 3.) United Gas Pipe Line Co. has finished its 204-mile natural gas pipe-line from the Lirette gas field in lower Louisiana to Mobile, on the coast of Alabama, a route which involved building the world's longest stretch of under water pipe and a number of river crossings, including a triple line under the Mississippi River. Constructed to supply the increasing demands for natural gas fuel by industries along the Gulf Coast from the Mississippi River eastward to Mobile and Pensacola, Fla., the pipeline was blown and tested at its Mobile terminus. Embodied in construction of this line for purposes of economy and speed of construction were some of the most difficult pipelining feats ever accomplished, including the world's longest underwater pipe line, the longest single "shove" ever made, miles upon miles of construction through swampland, numerous river crossings. First notable feat was on the Mississippi, 12 miles above New Orleans where the crossing was laid in seven days. The river is 2800 feet wide and approximately 90 to 120 feet deep with a very swift current. The greatest part of the job came when the line was laid across Lake Pontchartrain, 25.59 miles wide at this point. Serious consideration had been given to a land route, skirting the western and northern shores of the lake. Twenty-five miles across the lake, as opposed to the 37 miles by land which would have been mostly swamp construction, won out. The lake crossing presented not only the problems of the proper use of marine equipment and a gamble with one of the country's most temperamental bodies of water, but presented a considerable saving in steel pipe. Pipe size of 14 inches with one half inch wall and an outer protective coating almost an inch thick was used for the lake crossing. Pipe was welded into lengths of approximately 2500 feet on shore and towed out into the lake by tugboats. Line was laid from the center of the lake into shore by means of two huge derrick barges and a welding barge. As each tow came up one end was lifted aboard the welding barge and welded to the pipe already laid in the lake. The equipment was then moved down to the lower end of this section where the next tow was welded up. This process was repeated until the lake crossing had been accomplished. It took only 45 days to do this job, unique in the annals of pipelining, in spite of thru storms of hurricane force during which it was necessary to move all equipment from the lake.

R. H. O.

Butyl Rubber May Become Most Suitable Material For Tires.—(*The Oil and Gas Journal*, Vol. 40, No. 45.) The use of butyl, newest and least expensive of the synthetic-rubber products derived from petroleum hydrocarbons, in the manufacture of whole tires—carcass, tread, and tube—may eventually solve the shortage in this country. This possibility was advanced recently by W. S. Farish, president of the Standard Oil Company (New Jersey). Standard of New Jersey is embarked on a program that will provide for the United States and Canada a total of more than 200,000 short tons of synthetic rubber and raw materials for buna rubber by the middle of 1943. "Fulfillment of military needs is, of course, the immediate necessity," Mr. Farish said in reviewing the oil industry's program. "Those needs," he continued, "must be taken care of from the stock pile of natural rubber, plus the output of synthetic-rubber units included in the program announced recently by Secretary of Commerce Jesse H. Jones, plus natural production still available to the United Nations. But after these needs are provided for the only limitation on supply of synthetic rubber for civilian use is shortage of fabricated steel and other construction materials required for the necessary plants. It is difficult to see how any substitute for rubber tires could be provided with an estimated investment cost of less than $15 per car for plants to make synthetic rubber. Synthetic rubber of the buna type has always cost at least twice as much as the natural product. For large scale operations including the raw materials needed, the investment required to produce buna rubber is $1000 per ton of yearly capacity." "A new type of synthetic rubber developed by us, known as butyl, can be produced for an investment of about $700 per ton of yearly capacity. This rubber has never been produced commercially, but we have made it for some time experimentally, and large samples have been tested in tires by two large rubber companies. There is good reason to hope that whole tires—carcass, tread, and tubes—can be made successfully from this synthetic rubber. These tires would not at once be comparable with the buna tires or the natural-rubber tires now in use, but they should give satisfactory service at speeds up to at least 35 m.p.h. and should last for 10,000 miles. The cost of this rubber is estimated to be about one-half of the cost of buna rubber."

R. H. O.